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An improved Tangent Search Algorithm

Probhat Pachung, Jagdish Chand Bansal

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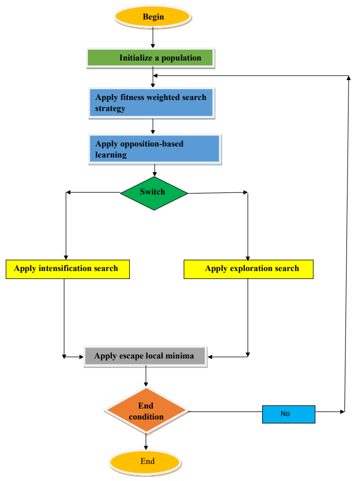
An improved Tangent Search Algorithm (iTSA)



Method Article - Title Page

Title	An improved Tangent Search Algorithm
Authors	Probhat Pachung[1], Jagdish Chand Bansal[2]
Affilliations	[1,2]South Asian University, New Delhi, India
Corresponding AuthorâĂŹs email address	jcbansal@sau.ac.in
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Abstract

The Tangent Search Algorithm (TSA) is a newly developed population-based meta-heuristic algorithm to solve complex optimization problems. It is based on the tangent function, which steers the given solution towards more promising regions of the search space. Though TSA has performed well for many optimization problems, the experimental analyses show that it suffers from the low exploration ability and slow convergence rate. This article proposes an improved TSA algorithm (iTSA). Using two concepts, 'Fitness Weighted Search Strategy' (FWSS) and 'Opposition Based learning' (OBL), iTSA is better in terms of exploration while maintaining the high convergence rate of TSA.

- Fitness weighted search strategy (FWSS) is used to increase the exploration ability of TSA.
- Opposition based learning (OBL) is used to increase the convergence speed of TSA.
- Together, OBL and FWSS into iTSA outperformed the classical TSA and other considered state-of-the-art algorithms.

The performance of the proposed iTSA is validated on two sets of test functions: CEC14 benchmark functions and a set of 21 well-known classical benchmark functions. The obtained results are compared with those obtained from the basic TSA and other considered state-of-the-art algorithms.

Specification Table

Subject Area	Computer Science
More specific subject area	Meta-heuristics Algorithm
Method name	Mathematical inspired algorithm
Name and reference of original method	Tangent search algorithm for solving optimization problems. https://link.springer.com/article/10.1007/s00521-022-06908-z
Resource availability	N/A

Background

Over the last few decades, meta-heuristic algorithms have become popular and robust optimization methods to deal with complex real-world optimization problems. The reason behind this is that the deterministic mathematical methods fail to provide a satisfactory solution to complex real-world problems which usually contain non-linear, discontinuous, and non-convex objective functions and may have many local optima [1, 2, 3]. Therefore, meta-heuristic algorithms have attracted a lot of attention from researchers [4, 5] to solve these kinds of problems.

Recently, a few mathematically inspired algorithms have been proposed. These kinds of meta-heuristics use geometric, arithmetic or analytic functions to direct the search process. Few mathematically inspired meta-heuristic algorithms are- Sine Cosine Algorithm (SCA) [6], Spherical Search Optimizer (SSO) [7], The Arithmetic Optimization Algorithm [8], Stochastic Fractal Search [9] etc.
In 2021, Abdesslam Layeb, developed a new mathematically inspired algorithm named as Tangent Search Algorithm (TSA) which is based on the tangent function ($tan(\theta)$) [10]. Since the tangent function varies from $-\infty$ to $+\infty$, this function provides TSA a great capacity to explore the search space and its periodicity helps to maintain a good balance between exploitation and exploration of the search process.
Though in the original TSA, exploitation and explorations are well maintained sometimes it is prone towards the local optima and has low convergence rate. Therefore, in order to further improve, we have incorporated the concepts of 'Fitness Weighted Search Strategy' (FWSS) [11] and 'Opposition Based Learning' (OBL) [12] in TSA to get an improved TSA (iTSA).

The original TSA, FWSS and OBL are described in the following sections.

Tangent Search Algorithm (TSA)

The TSA algorithm was developed in 2021 by Abdesslem Layeb [10]. It is one of the newly developed mathematically inspired meta-heuristic algorithm to solve optimization problems. The algorithm is based on the mathematical tangent function. The tangent function has the capacity to explore the search space very well. Its variation between $-\infty$ to $+\infty$ and its periodicity assists to maintain a balance between intensification and exploration. The main steps of TSA are as follows:

Initialization

Like any other population-based optimization algorithm, TSA generates a uniformly distributed random initial population over the search space by using the following equation:

$$X^0 = l + (u - l) * rand(). \tag{1}$$

Where l denotes the lower bound of the search space, u denotes the upper bound of the search space, and $rand()$ generates a uniformly distributed random numbers between 0 and 1.

After initialization, the following three steps are executed:
1. Intensification search
2. Exploration search and
3. Escape local minima procedure.
The detailed explanations of these steps are as follows:

Intensification Search

It is also called an exploitation search component and is used to guide the search process towards the optimal solution in the neighborhood of the current solution. At any iteration t, an individual takes a random local walk lead by Equation (2). Then, 20% variables of the attained solution are changed with the values of the corresponding variables in the current best solution using Equation (3) for the problem having dimensions greater than 4. While the variables of the obtained solution are changed by 50% for the problem having dimensions less or equal to 4. As a result, the newly obtained solution has a similarity rate of less than 50% with the optimal current solution, which helps to increase the existing solution locally.

$$X_i^{t+1} = X_i^t + stepsize1 * \tan(\theta) * \left(X_i^t - X_{best}^t\right), \forall i = 1, 2, \ldots, N. \tag{2}$$

$$X_i^{t+1} = X_{best}^t, \text{ if variable } i \text{ is selected }. \tag{3}$$

Where N denote the population size, X_i^t is the i^{th} solution of the current population, X_{best}^t is the best solution of the current population, stepsize1 is a parameter and is calculated as below:

$$stepsize1 = 10*signum\,(rand - 0.5)*norm\left(X_{best}^t\right)*\log\left(1 + 10 * \frac{D}{t}\right). \quad (4)$$

Where $norm()$ is the mathematical Euclidian norm and D is the dimension of the problem. Since in Equation (2), $\theta = \frac{\pi}{2}$ will make the algorithm diverge, so θ is randomly chosen from the range $[0, \frac{\pi}{2.1}]$.
Moreover, if the values of the obtained solution exceed the problem bounds i.e. l and u, then it is corrected by Equation (5).

$$X = rand * (u - l) + l; \quad (5)$$

Exploration Search

The exploration component helps to explore the search space to find the most promising domains. In the exploration search, a global random walk is made using a product of variable step-size and tangent flight. The tangent function assists in exploring the search space efficiently. In fact, θ near $\frac{\pi}{2}$ will increase the tangent value too much, resulting in a far different solution than the current one. Meanwhile, θ near 0 will give the small values to the tangent function resulting in a much closer solution to the current solution. Therefore, Equation (6) ensures that a sufficiently large part of the search space is covered during the exploration search. By using exploration search Equation (6), TSA combines global and local random walks.

$$X_i^{t+1} = X_i^t + stepsize2 * \tan(\theta) \quad (6)$$

Where $stepsize2$ is a parameter and is calculated as below:

$$stepsize2 = 1 * signum\,(rand - 0.5) * \frac{norm(optX^t - X_i^t)}{log(20 + t)} \quad (7)$$

Where $norm()$ is the conventional Euclidean norm and the parameter θ is chosen randomly from the range $[0, \frac{\pi}{3}]$, while the other symbols have their usual meanings as described above.
In the optimization method, the intensification and exploration search processes are performed based on a switching parameter $Pswitch \in [0, 1]$(see Algorithm 1).

Local Minima Escape Procedure

In order to escape from the local minima, TSA uses the local minima escape procedure after intensifying and exploring the search space. It has two parts which are executed with a probability $Pesc \in [0, 1]$. In each iteration t, a solution is chosen randomly, and then one of the following two equations (8) or (9) is applied based on Algorithm 2.

$$X_i^{t+1} = X_i^t + R. * \left(optX^t - rand * \left(optX^t - X_i^t\right)\right), \text{ if } i^{th} \text{ agent is selected.} \tag{8}$$

$$X_i^{t+1} = X_i^t + \tan(\theta) * (ub - lb) \tag{9}$$

Where $R = 10 * \frac{sign(0.5 - rand)}{log(1+t)}$ and $\theta = rand * \pi$ and other symbols have their usual meaning.

Algorithm 1 The pseudo code of TSA.

```
while t ≤ MAX_F ES do                                   ▷ Maximum iteration
    for each search agent X_i, i = 1 : N, do            ▷ N :Population size
        if rand < Pswicth then
            Apply intensification search(as in Equation 2);
        else
            Apply exploration search(as in Equation 6);
        end if
    end for
    if rand < Pesc then
        Randomly choose an agent;
        apply local minima escape procedure(as in Equation 8 or 9 and using Algorithm 2);
    end if
    increase iteration by one;
end while
```

Parameters Explanation

In contrast to many optimization algorithms, TSA uses fewer parameters. In TSA, the major parameters are $Pswitch$, $Pesc$, $stepsize1$ & $stepsize2$ and θ and these parameters are used to stress the exploitation and exploration of

Algorithm 2 The pseudo code of the escape local procedure.

Insert solution X_{old};
Generate step size $R1 = 10 * \frac{\text{sign(0.5-rand)}}{\log(1+t)}$;
if $rand_1 \leq 0.99$ **then**
  if $rand_2 < 0.8$ **then**
    $X_{new} = X_{old} + R1. * (optX - rand * (optX - X_{old}))$; (Equation 8)
  else
    $X_{new} = X_{old} + \tan(\theta) * (u - l)$; (Equation 9)
  end if
else
  Replace X_{old} by a new solution generated by using Equation 1.
end if
Output new solution X_{new}.

the search process. The parameter $Pswitch \in [0, 1]$, is used to manage the balance between local and global random walks. The parameter $Pesc \in [0, 1]$, represents the probability of escape from local minima. $step1$ and $step2$ are step size parameters which are used to direct and emphasize the search process. Variable step sizes are used in TSA to obtain a good estimate of the best solution and keep away from deficiency of accuracy. At the initial stage, TSA starts with large step size, but as iterations progress, the size of the step decreases non-linearly after each iteration. This adaptive behavior of the step sizes allow TSA to maintain a good balance between exploitation and exploration. Also, step sizes are highly affected by oscillatory and periodic behavior of the tangent function. To embrace the exploitation and intensification search process, TSA uses the logarithm function for the adaptive step-sizes. As a slow function, the logarithm function assists TSA in maintaining a good convergence rate. Further, the use of different step sizes in an algorithm leads to better results, especially for functions with hard convergence. This motivation led TSA to utilize two different step sizes. In intensification search, the first variant of step size is used. It is calculated as in Equation (4), and the second variant of step size is used in exploration search which is calculated as in Equation (7). Here, the exploitation and exploration search is directed by the component $sign(-, +)$. Furthermore, the parameter θ represents an angle that is important in determining the convergence of the TSA algorithm.

Fitness Weighted Search Strategy (FWSS)

The concept of the Fitness Weighted Search Strategy (FWSS) was first proposed by Yuksel Celik in 2021 to improve the search mechanism of the ABC algorithm [11]. In the original ABC, a new value is obtained between the current food source and one randomly selected food source to move towards a good food source. Since the direction of the new value obtained is random, there is no control over it whether the direction of the new value is good or bad. This has an overall negative impact on the performance of the original ABC algorithm. To overcome this drawback, FWSS uses two search spaces that are combined in two different directions, where each search space has a length equal to the original ABC search length. Also, a new value is determined by considering the weight of the fitness value as in Equation (10). In Equation (10), if the fitness value of the current solution is better than the random solution, then the direction of the new value is towards the current solution. Similarly, if the converse happens, the direction of the new value is given towards the random solution. Thus, by considering the fitness values of the current and the random solutions, the search space of the original ABC becomes more diversified, and the negativity of the algorithm can be mitigated.

$$w_{i,j} = \begin{cases} \frac{(fit_i * x_{i,j} - fit_k * x_{k,j})}{(fit_i + fit_k)}, & \text{if } fit_i > fit_k, \quad i = 1, \ldots, N \;\&\; j = 1, \ldots, D \\ \frac{(fit_i * x_{i,j}) + (fit_k * x_{k,j})}{(fit_i + fit_k)}, & \text{if otherwise.} \end{cases} \tag{10}$$

Where k is the randomly selected neighbor's index, N is the size of the population, D is the dimension of the problem, $x_{i,j}$ denotes the j^{th} variable of the i^{th} solution of the current population, fit_i denotes the fitness value of the i^{th} solution of the current population, fit_k is fitness value of the randomly selected neighbor's solution and finally, $w_{i,j}$ denotes the newly obtained j^{th} variable of the i^{th} weighted solution.

Opposition Based Learning (OBL)

Opposition-Based Learning (OBL), a new concept in machine intelligence introduced by Tizhoosh in 2005 [12]. It is based on the concept of considering an estimate and at the same time its corresponding opposite estimate in order to accelerate the solution-finding process. Now the opposite number for an $x_{ac} \in [a, b]$ is defined as the number $a + b - x_{ac}$.

[12] Similarly, for a D- dimensional vector $X_{ac} = (x_1, x_2, ..x_D) \in R^D$, where $x_i \in [a_i, b_i], i = 1, 2, \ldots, D$, the opposite point $\tilde{X_{op}} = (\tilde{x_1}, \tilde{x_2}, ..\tilde{x_D}) \in R^D$ is obtained by the following equation-

$$\tilde{x}_j = a_j + b_j - x_j \qquad j = 1, 2, \ldots D. \tag{11}$$

In opposition based optimization, for each D dimensional solution X, its opposite solution $\tilde{X}$ is computed simultaneously as in Equation (11). Now the greedy selection approach is used to select the solution between X, and $\tilde{X}$, i.e. if $f(.)$ is the fitness function, then the solution $\tilde{X}$ is chosen if $f(\tilde{X})$ has a better fitness value than $f(X)$; otherwise, X is chosen.

This article utilizes FWSS to increase the exploration ability of the search mechanism of basic TSA, while OBL is used to accelerate convergence speed.

Method details

Proposed Method

As we know, the performance of any existing algorithm may be improved by using two approaches: improving the existing search technique or developing a new search technique and hybridizing it with different algorithms [13, 14, 15, 16, 17, 18]. This article belongs to the first approach. Here we have modified the position update equation of TSA to improve the performance of the basic TSA.

As mentioned above, TSA uses a fewer number of parameters which are used to emphasize the exploitation and exploration search processes. Also, it is capable of obtaining a good balance between exploitation and exploration search. However, in some cases, the experiments have demonstrated that TSA is more inclined to local optima because of the inadequate exploration ability of the search agents. Therefore, in this article, the above-defined two strategies (FWSS and OBL) have been used to modify the basic TSA in expectation of better performance in terms of exploration and convergence speed.

The applied two strategies are discussed below.

Firstly, inspired by FWSS, we intelligently obtain a new weighted value ($W_i = (w_{i,1}, \ldots, w_{i,D})$) corresponding to each solution($X_i = (x_{i,1}, \ldots, x_{i,D})$) of the current population as in Equation (12).

$$w_{i,j} = \frac{fit_i * x_{i,j} + Bestfit * Bestposition_j}{fit_i + Bestfit}, \qquad i = 1, \ldots, N \ and \ j = 1, \ldots, D \tag{12}$$

Where N is the size of the population, D is the dimension of the problem, $x_{i,j}$ is the j^{th} variable of the i^{th} solution of the current population, $Bestfit$ is the fitness value of the best solution for the current population and $Bestposition_j$ is the position value of the j^{th} variable of the best solution in the current population, fit_i is the fitness value of the i^{th} solution of the current population and finally, $w_{i,j}$ represents the newly obtained j^{th} variable of the i^{th} weighted solution(W_i).

In Equation (12), the new weighted value is generated around the best solution which helps to enhance the exploration.

In addition to this, in expectation of better convergence, the concept of OBL is employed on the obtained weighted value (W_i) to get an opposite weighted value ($\tilde{W}_i = \tilde{w}_{i,1}, \ldots, \tilde{w}_{i,D}$) as in the following Equation (13).

$$\tilde{w}_{i,j} = l + u - w_{i,j} \qquad i = 1, \ldots, N, \ and \ j = 1, \ldots, D \tag{13}$$

Where l and u are the lower and upper bounds of the decision variable, N is the size of the population, D is the dimension of the problem, $w_{i,j}$ is the j^{th} variable of the i^{th} weighted value, and finally, $\tilde{w}_{i,j}$ is the j^{th} variable of the i^{th} opposite weighted value.

Finally, the current solution (X_i) in the position update equation of basic TSA is replaced by a new solution selected between the newly generated weighted value (W_i) and the opposite weighted value ($\tilde{W}_i$). In this selection, the greedy selection approach is used as in Equation (14).

$$X_i = \begin{cases} W_i, & \text{if } fit_{W_i} > fit_{\tilde{W}_i} \\ \tilde{W}_i, & \text{otherwise} \end{cases} \tag{14}$$

Where fit_{W_i} is the fitness value of the newly generated weighted value W_i and $fit_{\tilde{W}_i}$ is the fitness value of the opposite solution of W_i.

After experimental analyses, it has been observed that by replacing the current solution X_i in the position update equation of the basic TSA algorithm with the weighted value as in Equation (14), the search agents become more capable of searching the most promising regions and obtaining a high convergence rate.

The steps of the proposed iTSA are presented in Algorithm (3) in the form of pseudo-code.

Method validation

Benchmark functions and parameter settings

In this article, unless stated, '*Dim*' denotes the dimension of the problem, '*Range*' denotes the range of the decision variable, '*N*' represents the nature of the function and '*U*' and '*M*' respectively denote the unimodal and multimodal functions and 'f_{min}' represents the optimal value of the problem. To study the performance of iTSA, two sets of benchmark functions: CEC14 [25] and a set of 21 well-known classical test functions are used. In this section, first, we will discuss the performance of the proposed iTSA on 21 classical test functions, and then we will further evaluate its performance over the CEC14 benchmark test functions. The classical test functions are presented in Tables 1, 2, 3. These functions are represented by $f_1,\ f_2,\ldots,f_{21}$. In Table 1, functions $f_1, f_2, \ldots, f_7$ are presented, which are unimodal benchmark functions. Therefore, these functions are used to determine the exploitation ability and the convergence speed of meta-heuristic algorithms.

In Tables 2 & 3, multimodal benchmark functions($f_8 - f_{21}$) are listed. There may be more than one local optima in these benchmark functions, making it difficult for them to be tackled. These functions are suitable for benchmarking the exploration ability of an algorithm.

Firstly, numerical results on these problems are obtained by using iTSA and the results are compared with those obtained using the basic TSA, two possible variants of TSA: TSA with OBL (TSAOBL), TSA with FWSS (TSAFWSS), and seven other state-of-the-art meta heuristic algorithms (Particle swarm optimization (PSO) [19], Artificial Bee Colony (ABC) algorithm [20], Differential Evolution (DE) [21], Grey Wolf Optimizer (GWO) [22],

Algorithm 3 The pseudo code of iTSA.

Initialize the parameters;
Initialize the positions of search agents say X_i, $(i = 1, , \ldots, popsize)$ using Equation 1;
Calculate the fitness $f(X_i)$ of X;
while $t < MAX_F ES$ **do**
  for $i = 1 : popsize$ **do**
    Evaluate new weighted value W_i(using Equation 12);
    Evaluate the fitness $f(W_i)$ of W_i;
    Evaluate the opposite solution $\tilde{W}_i$ of W_i(using Equation 13);
    Evaluate the fitness $f(\tilde{W}_i)$ of $\tilde{W}_i$;
    if $f(W_i) < f(\tilde{W}_i)$ **then**
      $X_i = W_i$;
    else
      $X_i = \tilde{W}_i$;
    end if
    if $rand < Pswitch$ **then**
      Apply intensification search(as in Equation 2);
    else
      Apply exploration search(as in Equation 6);
    end if
  end for
  if $rand < Pesc$ **then**
    Select and agent search randomly;
    Apply escape local minima procedure(using Algorithm 2);
  end if
  increase iteration by one;
end while

Table 1
Unimodal test functions.

Function	Name	Dim	Range	Nature (Unimodal)	f_{min}
$f_1(x) = \sum_{i=1}^{n} {x_i}^2$	Sphere function	30	$[-100, 100]$	U	0
$f_2(x) = \sum_{i=1}^{n} \mid x_i^2 \mid + \prod_{i=1}^{n} \mid x_i \mid$	Schwefel's problems 2.22	30	$[-10, 10]$	U	0
$f_3(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j \right)^2$	Schwefel's problems 1.2	30	$[-100, 100]$	U	0
$f_4(x) = \max_i \{\mid x_i \mid, 1 \leq i \leq n\}$	Schwefel's problems 2.21	30	$[-100, 100]$	U	0
$f_5(x) = \sum_{i=1}^{n-1} \left[100 \left(x_{i+1} - {x_i}^2 \right)^2 + (x_i - 1)^2 \right]$	Generalized Rosenbrock's funciton	30	$[-30, 30]$	U	0
$f_6(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$	Step function	30	$[-100, 100]$	U	0
$f_7(x) = \sum_{i=1}^{n} i{x_i}^4 + random[0, 1)$	Quartic function with noise	30	$[-1.28, 1.28]$	U	0

Table 2
Multimodal test functions.

Function	Name	Dim	Range	Nature (Multimodal)	f_{min}
$f_8(x) = \sum_{i=1}^{n} \left[{x_i}^2 - 10\cos(2\pi x_i) + 10 \right]$	Generalized Rastrigin function	30	[-5.12,5.12]	M	0
$f_9(x) = -20exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} {x_i}^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	Ackley's function	30	[-32,32]	M	0
$f_{10}(x) = \frac{1}{4000}\sum_{i=1}^{n} {x_i}^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	Generalized Griewank function	30	[-600,600]	M	0
$f_{11}(x) = \frac{\pi}{n}\left\{10sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right]\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$, $y_i = 1 + \frac{x_i + 1}{4}$, $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m x_i & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m x_i & x_i < -a \end{cases}$	Generalized penalized function	30	$[-50, 50]$	M	0
$f_{12}(x) = 0.1\{\sin^2(3\pi x_1) + \sum_{i=1}^{n}(x_i - 1)^2\left[1 + \sin^2(3\pi x_i + 1)\right] + (x_n - 1)^2\left[1 + \sin^2(2\pi x_n)\right]\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	Generalized penalized function	30	[-50,50]	M	0

Table 3
Fixed-dimensional multimodal test function.

Function	Name	Dim	Range	Nature (Multimodal)	f_{min}
$f_{13}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j+\sum_{i=1}^{2}(x_i - a_{ij})^6}\right)^{-1}$	Shekel's Foxholes function	2	[-65,65]	M	0.998
$f_{14}(x) = 4{x_1}^2 - 2.1{x_1}^4 + \frac{1}{3}{x_1}^6 + x_1x_2 - 4{x_2}^4$	Six-hump camel-back function	2	[-5,5]	M	-1.0316
$f_{15}(x) = \left(x_2 - \frac{5.1}{4\pi^2}{x_1}^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos x_1 + 10$	Branin function	2	[-5,5]	M	0.398
$f_{16}(x) = \left[1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3{x_1}^2 - 14x_2 + 6x_1x_2 + 3{x_2}^2)\right] \times \left[30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12{x_1}^2 + 48x_2 - 36x_1x_2 + 27{x_2}^2)\right]$	Goldstein -Price function	2	[-2,2]	M	3
$f_{17}(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2\right)$	Hartman's family function	3	[1,3]	M	-3.86
$f_{18}(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2\right)$	Hartman's family function	6	[0,1]	M	-3.32
$f_{19}(x) = -\sum_{i=1}^{5}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	Shekel's family function	4	[0,10]	M	-10.1532
$f_{20}(x) = -\sum_{i=1}^{7}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	Shekel's family function	4	[0,10]	M	-10.4028
$f_{21}(x) = -\sum_{i=1}^{10}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	Shekel's family function	4	[0,10]	M	-10.5363

Sine-Cosine Algorithm (SCA) [6], Salp-Swarm Algorithm (SSA) [23], CMA-ES [24]. For all the considered algorithms, the maximum number of iterations and the swarm size are set to be 2000 and 30, respectively. Moreover, each algorithm is tested on 30 individual runs, and the results are captured. For all the experiments in this article, the parameter settings are as follows:

1. iTSA : $Pswitch = 0.3,\ Pesc = 0.8$
2. TSA : $Pswitch = 0.3,\ Pesc = 0.8$
3. GWO : Same as in [22]
4. PSO : $c_1 = c_2 = 1.47,\ w = 0.7$
5. DE : $F = 0.5,\ P_{\mathrm{cr}} = 0.7$

6. ABC : Same as in [20]
7. SCA : Same as in [6]
8. SSA : Same as in [23]
9. CMA-ES : Same as in [24]

Comparison of improved TSA(iTSA) with basic TSA, TSAFWSS and TSAOBL

Here, the numerical results obtained in the iTSA, the basic TSA, TSAFWSS and TSAOBL are recorded and are presented in Tables 4, 5, and 6. Here, '$Mean$' indicates the average value of the optimal values, '$Best$' and '$Worst$' refer to the best and the worst value obtained in the 30 independent runs for each function, '$Median$' denotes the median value of the optimal values, and 'SD' represents the standard deviation from the mean of the optimal values recorded over 30 individual runs. The best obtained value for each function among the algorithms are highlighted with boldface in Tables 4, 5 and 6.

Comparison in terms of exploitation and exploration

The exploitation and exploration ability in stochastic meta-heuristic algorithms can be determined by unimodal and multimodal benchmark functions, respectively. Considering the results in Table 4, we can say that for all the unimodal test functions except functions f_7 and f_5, iTSA performs better than TSA, TSAFWSS and TSAOBL in terms of Mean, Best, Worst, Median and SD, while for function f_7, the best value is obtained by TSAFWSS and for function f_5, TSAOBL outperformes TSA, iTSA and TSAFWSS in terms of Mean, Worst, and SD. These results demonstrates the better exploitation ability of iTSA than TSA, TSAFWSS and TSAOBL.

In Table 5, for test functions, f_9, f_{11}, f_{12}, iTSA performs better than TSA in terms of Mean, Best, Worst, Median and SD, while TSA, iTSA and TSAOBL perform equally for the test functions f_{10}; for function f_8, iTSA and TSAOBL both performs better in terms of Mean, Best, Worst, Median and SD. These results demonstrate that iTSA has a better exploration ability to find the most promising regions than that of the classical TSA, TSAFWSS and TSAOBL.

Table 6 illustrates that iTSA performs equally with TSA in terms of Mean, Best, Worst, Median for the test problems f_{13}, f_{15}, f_{17}, f_{19}, f_{20} and

f_{21}, which demonstrates that iTSA performs equally with TSA in the case of solving fixed dimensional multimodal problems also.

It can be observed from the above discussions that OBL and FWSS with TSA are unable to outperform basic TSA when they are independently present with TSA i.e. TSAOBL and TSAFWSS.

Hence, the overall comparisons of numerical results based on Mean, Median, Standard Deviation, Best and Worst of the optimal values over 30 independent runs for each problem confirms that the proposed iTSA outperforms the classical TSA, TSAFWSS and TSAOBL for the unimodal and multimodal test problems and performs equally for fixed-dimensional multimodal test functions.

Comparison of improved TSA(iTSA) with well-known meta-heuristic algorithms on classical test problems

In this section, the performance of the proposed iTSA algorithm is compared with other considered state-of-the-art meta-heuristics algorithms (GWO, PSO, DE, ABC, SCA, SSA,CMA-ES). During the experiments, the same benchmark sets (Tables 1, 2, and 3) are used. The same parameter settings and under the same experimental environment as given above are used. The comparison of results between iTSA and other considered meta-heuristics algorithms are shown in Tables 7, 8, 9 and 10.

Based on mean, iTSA performs better for all functions $f_1 - f_{21}$ except for 6 functions (f_4, f_6, f_{11}, f_{12}, f_{16}, and f_{18}). For function f_{14}, the mean values are the same for all algorithms, while for function f_{15}, all algorithms have equal mean values except SCA. When best values are taken into account, iTSA outperforms for all functions $f_1 - f_{21}$ except for the functions f_6, f_{11} and f_{12}. For functions f_{14}, f_{15}, f_{16}, and f_{17}, the minimum values are the same in all the algorithms and for functions f_{19}, f_{20} and f_{21}, except SCA all algorithms perform equal best values. Out of 21 functions, iTSA performs better on functions 15, 11, and 12 based on Median, Standard Deviation(SD) and Worst values, respectively.

In addition, to show the empirical distributions of data, the box plots are given in Figure 1. Here the box plots are plotted for Mean, Best, Standard Deviation, Median, Worst, and number of functions evaluations corresponding to all algorithms.

Thus from the above discussions, one can prefer iTSA to obtain a better

Table 4
The results obtained by iTSA, classical TSA, TSAFWSS and TSAOBL on unimodal test problems.

Function	Algorithms	Mean	Best	Worst	Median	SD
f_1	TSA	8.8246E-24	1.7911E-29	8.2973E-23	6.2651E-26	2.2377E-23
	iTSA	**9.294E-162**	**5.659E-178**	**2.674E-160**	**3.217E-169**	**4.875E-161**
	TSAFWSS	3.06299E-23	1.45592E-27	4.05094E-22	8.02255E-25	1.03814E-22
	TSAOBL	7.59E-29	9.25723E-33	7.6822E-28	6.05639E-30	1.97002E-28
f_2	TSA	5.4292E-14	1.5073E-16	4.3413E-13	1.0356E-14	1.0835E-13
	iTSA	**2.40E-115**	**1.542E-120**	**2.739E-114**	**1.133E-116**	**5.636E-115**
	TSAFWSS	2.9404E-14	1.28434E-16	2.18496E-13	1.05429E-14	5.55217E-14
	TSAOBL	6.08E-18	1.84321E-19	3.31451E-17	7.73696E-19	9.33382E-18
f_3	TSA	2.0124E-09	2.3753E-16	3.9457E-08	7.834E-12	7.6275E-09
	iTSA	**1.7044E-43**	**1.3689E-62**	**4.2124E-42**	**9.4918E-51**	**7.7333E-43**
	TSAFWSS	3.36E-09	9.74572E-15	4.99934E-08	9.9944E-12	1.2901E-08
	TSAOBL	3.33E-08	2.62013E-11	2.5889E-07	4.53501E-09	7.46471E-08
f_4	TSA	0.00391879	1.387E-05	0.01656141	0.00110256	0.00493157
	iTSA	**6.9632E-10**	**1.984E-101**	**1.9143E-08**	**7.9945E-12**	**3.4862E-09**
	TSAFWSS	0.004841297	1.19581E-05	0.01972914	0.001020922	0.006486786
	TSAOBL	0.0087683390.00	0158499	0.021887745	0.00677969	0.006707629
f_5	TSA	4.4334826	1.5852E-07	20.2603014	0.00019538	8.1889492
	iTSA	1.51650747	**2.366E-10**	17.8170721	**7.5207E-06**	4.66173745
	TSAFWSS	1.144533488	6.05207E-08	16.86081079	4.12498E-05	4.348059518
	TSAOBL	**9.91606E-05**	2.47812E-06	**0.000664218**	1.53571E-05	**0.000204238**
f_6	TSA	2.1272E-23	1.379E-26	1.4704E-22	7.2831E-24	3.7448E-23
	iTSA	**1.4733E-28**	**6.163E-33**	**2.0718E-27**	**3.5591E-29**	**3.8121E-28**
	TSAFWSS	4.78571E-23	2.57213E-25	2.12817E-22	2.02318E-23	6.08957E-23
	TSAOBL	3.51334E-22	4.5101E-25	4.9186E-21	1.13845E-23	1.26412E-21
f_7	TSA	0.00377836	0.00024328	0.0151328	0.00285138	0.0035145
	iTSA	**0.00039039**	8.3896E-05	**0.0009129**	**0.00036443**	**0.00020098**
	TSAFWSS	0.002290967	**6.89983E-06**	0.00776599	0.001719429	0.001960937
	TSAOBL	0.000801095	5.7511E-05	0.001757443	0.000865812	0.000532402

Table 5
The results obtained by iTSA, classical TSA, TSAFWSS and TSAOBL on multimodal test problems.

Function	Algorithms	Mean	Best	Worst	Median	SD
f_8	TSA	9.796E-13	**0**	2.8933E-11	**0**	5.2798E-12
	iTSA	**0**	**0**	**0**	**0**	**0**
	TSAFWSS	0.066330604	**0**	0.994959057	**0**	0.256897324
	TSAOBL	**0**	**0**	**0**	**0**	**0**
f_9	TSA	6.0852E-13	7.9936E-15	8.1011E-12	1.8563E-13	1.4596E-12
	iTSA	**3.3751E-15**	8.8818E-16	**4.4409E-15**	4.4409E-15	**1.6559E-15**
	TSAFWSS	3.96E-13	**8.88178E-16**	2.18581E-12	2.24709E-13	5.56611E-13
	TSAOBL	6.09883E-15	**8.88178E-16**	1.5099E-14	**4.44089E-15**	3.51872E-15
f_{10}	TSA	**0**	**0**	**0**	**0**	**0**
	iTSA	**0**	**0**	**0**	**0**	**0**
	TSAFWSS	2.22045E-17	**0**	3.33067E-16	**0**	8.59975E-17
	TSAOBL	**0**	**0**	**0**	**0**	**0**
f_{11}	TSA	2.6722E-22	4.3161E-26	1.7712E-21	4.9682E-23	5.2893E-22
	iTSA	**1.0151E-24**	**4.6947E-30**	**2.1119E-23**	**6.4202E-26**	**3.8478E-24**
	TSAFWSS	2.43199E-21	1.06834E-23	2.95116E-20	2.37956E-22	7.51692E-21
	TSAOBL	1.99244E-22	2.84917E-25	2.14906E-21	5.57827E-24	5.47198E-22
f_{12}	TSA	0.00109874	5.3673E-24	0.01098737	6.3412E-22	0.00335256
	iTSA	0.00073249	**1.1388E-29**	0.01098737	**7.205E-25**	0.00278758
	TSAFWSS	**1.87106E-21**	2.65375E-24	**7.18159E-21**	7.85428E-22	**2.40755E-21**
	TSAOBL	0.000732491	3.43794E-23	0.010987366	6.8685E-22	0.002836926

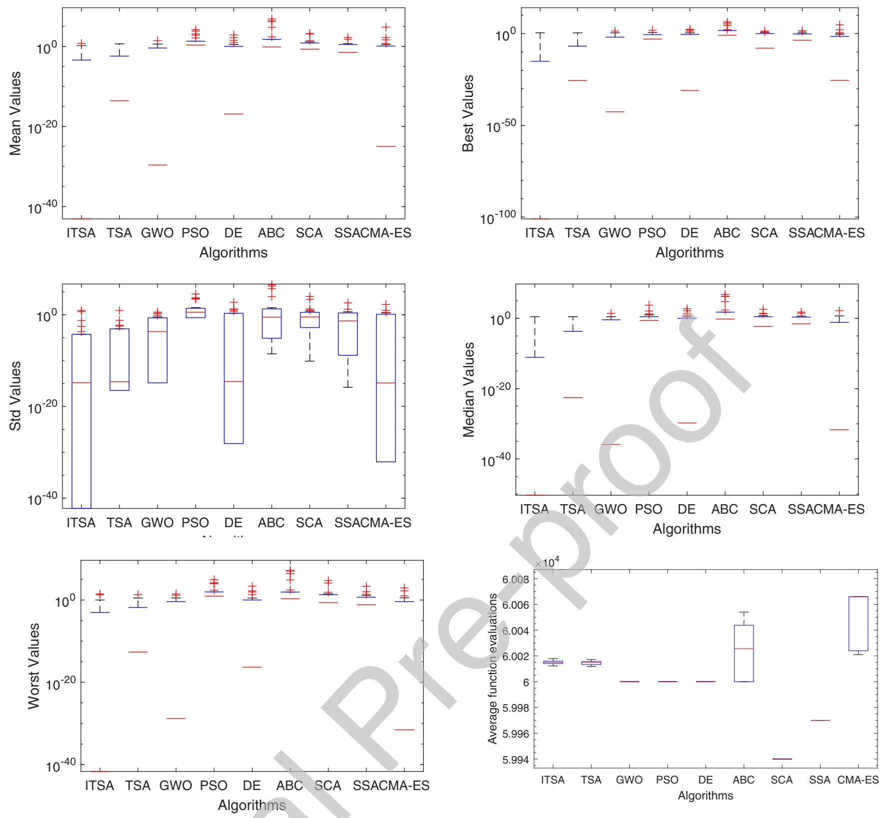

Figure 1: Box plots for the results of the meta-heuristic algorithms.

objective value in the 21 standard classical benchmark functions over other considered algorithms.

Convergence Analysis

In order to determine the convergence speed of the proposed algorithm, iTSA, it is compared with basic TSA and other considered meta-heuristics algorithms on classical benchmark test problems. The convergence curves are plotted between iterations and the optimal values obtained in each iteration. In the curves, the horizontal axis and the vertical axis represent the iterations of an algorithm and the objective function values, respectively. nine selected convergence curves corresponding to classical benchmark functions are presented in Figures 2 and 3. From Figures 2 and 3, it can be easily seen that iTSA has better convergence rate than all the other considered algorithms and has capability to find out the theoretically optimal solutions on functions f_1, f_2 and f_3. Also, iTSA can reach the global optima with the minimum number of iterations and has the fastest convergence rate just after GWO when solving the problems f_8, f_9, and f_{10}. Moreover, when solving the problems f_{15}, f_{16} and f_{17}, iTSA performs equally to the other algorithms. Thus, based on the above statistical results (Tables 4-9) and convergence curves (Figures 2 and 3), we can conclude that iTSA provides satisfactory performance in both solution accuracy and convergence rate. Hence, we can conclude that the comprehensive performance of iTSA in terms of exploitation and exploration for the considered test functions is better than that of TSA, TSAFWSS, TSAOBL, GWO, PSO, DE, ABC, SCA, SSA and CMA-ES.

Wilcoxon Signed-Rank Test

To further show the superiority of iTSA, we have performed Wilcoxon signed-rank test in classical benchmark test problems presented in Tables 13 and 14. In this non-parametric test, the median of the the best fitness values on classical benchmark test problems are taken for the comparison. The test is conducted with a significance level of 5% and the null hypothesis is that "There is no significant difference between iTSA and other algorithms". Here the value of h=0 and 1 indicates the rejection of the null hypothesis at 5% level of significance and failure to deny the null hypothesis, respectively. As we can see from Table 13; for the unimodal and multimodal classical test problems, iTSA behaves significantly different than TSA, GWO, PSO, DE, ABC, SCA and SSA algorithms with the p-values 9.7656E-04, 9.7656E-04, 7.3242E-04, 0.0161, 0.0215, 2.4414E-04 and 2.4414E-04, respectively. While in the case of fixed dimensional multimodal classical test problems (Table 9), the results of iTSA do not differ significantly from those of other algorithms considered. Thus, from the test results, we can conclude that iTSA is significantly superior to all other considered algorithms in terms of unimodal and multimodal classical benchmark problems.

Comparison of improved TSA with original TSA and other state-of-the-art algorithms on CEC14 test functions

To further show the superiority of the proposed iTSA, the CEC14 [25] benchmark test functions are used for evaluating the performance of the proposed iTSA based on the performance indicators: mean, best, median, worst, and standard deviation. The CEC14 [25] benchmark test function consists of 30 different types of functions, including 3 unimodal, 13 simple multimodal, 6 hybrid, and 8 composition functions. The performance of the proposed iTSA is compared with the basic TSA [10], TSAFWSS, TSAOBL, and six other state-of-the-art algorithms such as GWO [22], PSO [19], ABC [20], SCA [6], SSA [23], and CMA-ES [24]. The parameter settings of the considered algorithms are the same as done in classical test functions, while the following parameters are common to all algorithms.

- Swarm size=51
- Maximum iteration=2000
- Dim=30
- Lower bound of the search space (lb)=-100
- Upper bound of the search space (ub)=100

In addition, for a fair comparison, 30 independent runs are carried out corresponding to each algorithm, and the best results are recorded and compared among considered algorithms. The obtained results are shown in Tables 10-12. In these tables, the best of the mean values are highlighted in boldface. The convergence curves of nine selected test functions are presented in Figure 4 and Figure 5. In these graphs, the horizontal axis represents the iteration number while the vertical axis represents the objective function values. In addition, to show the distribution of the results, box plots for mean, best, median, worst, and standard deviation are presented in Figure 6. In Tables 10-12, when the proposed iTSA is compared with the original TSA in terms of mean values, it can be easily seen that out of 30 test functions, iTSA performs better in 18 test functions, including 3 unimodal, 8 simple multimodal, 3 hybrid, and 4 composition function while when it is compared with other considered algorithms, iTSA performs better than GWO, PSO, ABC, SCA, but CMA-ES and SSA shows very competitive results with that of proposed iTSA. The overall analysis of all these results concludes that the proposed iTSA performs better in unimodal and multimodal test functions over other considered algorithms. Thus one can prefer the proposed iTSA in unimodal and multimodal test functions as a better global optimization method.

Conclusion

In this article, an improved version of the basic TSA, namely the improved TSA (iTSA) algorithm has been proposed. The improvement is performed in the search process of basic TSA by using two strategies: Fitness Weighted Search Strategy (FWSS) and Opposition-Based Learning (OBL). Utilizing both strategies in the search space is meant to maintain the balance between the intensification and exploration of the search space while keeping the high convergence rate. The effectiveness of the proposed improved TSA is validated on CEC14 benchmark test problems and a set of 21-standard classical test functions. The obtained results are compared to those obtained by the original TSA and other considered state-of-the-art meta-heuristic algorithms. From the experimental results, it is found that the proposed iTSA provides a better exploration ability and is capable of maintaining a high convergence rate as compared to other considered state-of-the-art algorithms.

Declarations of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Table 6
The results obtained by iTSA, classical TSA, TSAFWSS and TSAOBL on Fixed-dimensional test problems.

Function	Algorithms	Mean	Best	Worst	Median	SD
f_{13}	TSA	**0.99800384**	**0.99800384**	**0.99800384**	**0.99800384**	4.1233E-17
	iTSA	**0.99800384**	**0.99800384**	**0.99800384**	**0.99800384**	4.1233E-17
	TSAFWSS	**0.99800384**	**0.99800384**	**0.99800384**	**0.99800384**	5.93439E-17
	TSAOBL	**0.99800384**	**0.99800384**	**0.99800384**	**0.99800384**	**0**
f_{14}	TSA	**-1.03162845**	**-1.03162845**	**-1.03162845**	**-1.03162845**	6.1849E-16
	iTSA	-1.0316253	-1.03162843	-1.03160756	-1.03162698	4.6215E-06
	TSAFWSS	**-1.03162845**	**-1.03162845**	**-1.03162845**	**-1.03162845**	**8.3925E-17**
	TSAOBL	-1.031628366	**-1.03162845**	-1.031628028	-1.031628391	1.11504E-07
f_{15}	TSA	**0.39788736**	**0.39788736**	**0.39788736**	**0.39788736**	**0**
	iTSA	**0.39788736**	**0.39788736**	**0.39788736**	**0.39788736**	**0**
	TSAFWSS	**0.39788736**	**0.39788736**	**0.39788736**	**0.39788736**	**0**
	TSAOBL	**0.39788736**	**0.39788736**	**0.39788736**	**0.39788736**	**0**
f_{16}	TSA	**3**	**3**	**3**	**3**	**1.716E-15**
	iTSA	5.7	**3**	30	**3**	8.23847157
	TSAFWSS	6.6	**3**	30	**3**	9.500375932
	TSAOBL	6.6	**3**	30	**3**	9.500375932
f_{17}	TSA	**-3.86278215**	**-3.86278215**	**-3.86278215**	**-3.86278215**	2.4959E-15
	iTSA	**-3.86278215**	**-3.86278215**	**-3.86278215**	**-3.86278215**	2.4945E-15
	TSAFWSS	**-3.86278215**	**-3.86278215**	**-3.86278215**	**-3.86278215**	8.92135E-15
	TSAOBL	**-3.86278215**	**-3.86278215**	**-3.86278215**	**-3.86278215**	**1.61869E-15**
f_{18}	TSA	**-3.26650747**	**-3.32199517**	**-3.20310205**	**-3.32193145**	0.06032427
	iTSA	-3.23480688	**-3.32199517**	**-3.20310205**	-3.20310205	**0.05347533**
	TSAFWSS	-3.234806883	**-3.32199517**	**-3.20310205**	-3.20310205	0.054421865
	TSAOBL	-3.242733091	**-3.32199517**	**-3.20310205**	-3.20310205	0.058013903
f_{19}	TSA	**-10.1531997**	**-10.1531997**	**-10.1531997**	**-10.1531997**	6.2847E-15
	iTSA	**-10.1531997**	**-10.1531997**	**-10.1531997**	**-10.1531997**	6.3278E-15
	TSAFWSS	**-10.1531997**	**-10.1531997**	**-10.1531997**	**-10.1531997**	1.1629E-15
	TSAOBL	**-10.1531997**	**-10.1531997**	**-10.1531997**	**-10.1531997**	**1.06158E-15**
f_{20}	TSA	**-10.4029406**	**-10.4029406**	**-10.4029406**	**-10.4029406**	**7.3759E-16**
	iTSA	**-10.4029406**	**-10.4029406**	**-10.4029406**	**-10.4029406**	9.8958E-16
	TSAFWSS	**-10.4029406**	**-10.4029406**	**-10.4029406**	**-10.4029406**	3.58429E-15
	TSAOBL	**-10.4029406**	**-10.4029406**	**-10.4029406**	**-10.4029406**	3.6156E-15
f_{21}	TSA	**-10.5364098**	**-10.5364098**	**-10.5364098**	**-10.5364098**	**1.1427E-15**
	iTSA	**-10.5364098**	**-10.5364098**	**-10.5364098**	**-10.5364098**	1.5116E-15
	TSAFWSS	**-10.5364098**	**-10.5364098**	**-10.5364098**	**-10.5364098**	1.64459E-15
	TSAOBL	**-10.5364098**	**-10.5364098**	**-10.5364098**	**-10.5364098**	1.3428E-15

Table 7
The results obtained by iTSA and other considered algorithms on unimodal test problems.

Function	Test	GWO	PSO	DE	ABC	SCA	SSA	CMA-ES	iTSA
	Mean	8.8E-122	1333.333	1.2E-30	0.31781	7.07E-09	7.7E-09	8.19E-53	**9.3E-162**
	Best	2.2E-124	2.44E-37	1.45E-32	0.012326	9.3E-13	4.22E-09	6.74E-54	**5.7E-178**
f_1	Worst	1.8E-120	10000	1.4E-29	2.147233	1.71E-07	1.17E-08	3.48E-52	**2.67E-160**
	Median	3.8E-123	7.73E-29	3.58E-31	0.129517	1.51E-10	7.23E-09	5.64E-53	**3.2E-169**
	SD	3.3E-121	3457.459	2.58E-30	0.487187	3.11E-08	1.96E-09	7.07E-53	**4.9E-161**
	Mean	1.47E-70	19.92122	2.62E-17	27.18242	4.68E-11	0.858358	2.1E-25	**2.4E-115**
	Best	3.64E-72	0.026945	4.79E-18	3.45E-05	4.57E-15	0.000536	6.66E-26	**1.5E-120**
f_2	Worst	1.34E-69	40.02822	9.26E-17	85.53748	3.09E-10	4.345363	4.26E-25	**2.7E-114**
	Median	2.99E-71	19.06709	2.3E-17	1.930436	5.39E-12	0.302614	1.86E-25	**1.1E-116**
	SD	2.91E-70	11.70413	1.8E-17	33.92407	7.67E-11	1.162925	1.08E-25	**5.6E-115**
	Mean	2.17E-33	6850.144	742.9266	57114.45	1316.179	3.746127	1.19775	**1.7E-43**
	Best	5.01E-43	0.261102	243.4192	31919.64	8.688929	0.093502	0.004974	**1.37E-62**
f_3	Worst	5.5E-32	15813.05	2201.769	75103.99	12561.89	22.98407	3.3129	**4.21E-42**
	Median	2.62E-36	5347.296	557.2358	57632.27	388.4161	2.339152	1.056117	**9.49E-51**
	SD	1E-32	5008.52	520.7358	9229.121	2367.632	4.365613	0.941648	**7.73E-43**
	Mean	**5.11E-30**	0.050031	8.454786	59.45566	7.177722	4.838114	2.17E-20	6.96E-10
	Best	2.74E-32	0.002218	0.247267	47.97468	0.138183	0.184928	6.65E-21	**2E-101**
f_4	Worst	**3.07E-29**	0.171682	20.01799	66.85551	19.15562	12.82004	5.07E-20	1.91E-08
	Median	**2.5E-30**	0.026677	7.855589	60.76005	5.802002	4.371126	2.13E-20	7.99E-12
	SD	**7.8E-30**	0.050793	5.873522	4.833122	5.970607	3.057143	1.03E-20	3.49E-09
	Mean	26.86926	15036.37	33.65044	1721567	28.24203	156.4531	43.0977	**1.516507**
	Best	25.18048	0.285631	24.0034	565541	26.73952	24.59717	0.031281	**2.37E-10**
f_5	Worst	28.73768	90045.03	82.71852	2458507	29.65086	2099.163	833.0849	**17.81707**
	Median	27.11432	15.65192	25.5522	1839418	28.17137	28.90346	0.07048	**7.52E-06**
	SD	0.81004	34105.36	18.77082	511256.6	**0.632552**	383.0795	171.0041	4.661737
	Mean	0.550855	660.0167	1.28E-30	0.234392	4.196311	7.81E-09	**0**	1.47E-28
	Best	6.52E-06	**0**	**0**	0.00536	3.620763	4.24E-09	**0**	6.16E-33
$f6$	Worst	1.095107	9900.25	5.89E-30	2.049696	5.145419	1.22E-08	**0**	2.07E-27
	Median	0.501179	1.56E-31	7.46E-31	0.09668	4.132926	7.8E-09	**0**	3.56E-29
	SD	0.326707	2511.774	1.58E-30	0.420718	0.326952	1.84E-09	**0**	3.81E-28
	Mean	0.000411	2.232278	0.009144	0.834875	0.011312	0.054628	0.00266	**0.00039**
	Best	0.000122	0.114181	0.003496	0.226227	0.001441	0.019706	0.000675	**8.39E-05**
f_7	Worst	**0.000892**	22.5407	0.021749	1.835374	0.035817	0.088265	0.00473	0.000913
	Median	0.000426	0.641446	0.008367	0.822319	0.009777	0.052178	0.00264	**0.000364**
	SD	0.000212	4.936388	0.003707	0.298369	0.007916	0.019557	0.000957	**0.000201**

Table 8
The results obtained by iTSA and other considered algorithms on multimodal test problems.

Function	Test	GWO	PSO	DE	ABC	SCA	SSA	CMA-ES	iTSA
f_8	Mean	**0**	132.9993	144.805	235.5569	7.778835	62.18481	153.0013	**0**
	Best	**0**	75.61665	124.4461	199.6236	5.36E-10	33.82854	127.621	**0**
	Worst	**0**	233.0323	164.4751	258.4921	74.92819	99.49564	167.0144	**0**
	Median	**0**	126.3591	145.3224	235.7816	0.000134	63.67726	154.9175	**0**
	SD	**0**	35.29643	11.85679	13.74708	18.14652	15.81979	9.10720	**0**
f_9	Mean	8.59E-15	10.35274	7.99E-15	0.699642	13.6419	2.298763	3.85E-15	**3.38E-15**
	Best	4.44E-15	4.382552	7.99E-15	0.04905	2.48E-08	0.931305	**8.88E-16**	**8.88E-16**
	Worst	1.51E-14	16.94193	7.99E-15	2.157847	20.29188	4.424418	**4.44E-15**	**4.44E-15**
	Median	7.99E-15	9.227311	7.99E-15	0.485303	20.15989	2.22097	**4.44E-15**	**4.44E-15**
	SD	1.89E-15	4.312116	**0**	0.602887	9.383463	0.741065	1.35E-15	1.66E-15
f_{10}	Mean	0.001063	6.221133	0.001807	0.778459	0.003754	0.012868	**0**	**0**
	Best	**0**	0.007396	**0**	0.251419	3.67E-12	1.67E-08	**0**	**0**
	Worst	0.013073	90.11043	0.014772	1.032968	0.046823	0.049091	**0**	**0**
	Median	**0**	0.095299	**0**	0.780484	2.11E-06	0.004929	**0**	**0**
	SD	0.003307	22.80076	0.004288	0.152349	0.011506	0.015496	**0**	**0**
f_{11}	Mean	0.031514	2.546792	8.45E-24	2217845	0.505454	4.128315	**1.57E-32**	1.02E-24
	Best	0.012507	5.94E-25	3.81E-32	45546.85	0.332532	0.726431	**1.57E-32**	4.69E-30
	Worst	0.077045	18.8319	2.53E-22	9747827	0.972097	11.66567	**1.57E-32**	2.11E-23
	Median	0.026584	1.405382	7.24E-31	1864844	0.452152	3.616304	**1.57E-32**	6.42E-26
	SD	0.016029	3.632915	4.62E-23	1791377	0.155342	2.645526	**5.57E-48**	3.85E-24
f_{12}	Mean	0.461497	2.224575	3.14E-29	7139884	1730.374	0.006895	**2.54E-32**	0.000732
	Best	0.101914	1.27E-25	1.4E-31	2560183	1.83715	3.87E-10	**1.35E-32**	1.14E-29
	Worst	0.91425	10.49266	5.4E-28	15636936	51843.34	0.043949	**4.43E-32**	0.010987
	Median	0.504256	1.126913	2.68E-30	6280296	2.311065	7.63E-10	**2.58E-32**	7.2E-25
	SD	0.169951	2.939616	1.07E-28	3592980	9464.828	0.00968	**1.1E-32**	0.002788

Table 9
The results obtained by iTSA and other considered algorithms on fixed-dimensional test problems.

Function	Test	GWO	PSO	DE	ABC	SCA	SSA	CMA-ES	iTSA
f_{13}	Mean	3.675984	1.493181	**0.998004**	0.998016	1.328731	0.998004	5.556	**0.998004**
	Best	**0.998004**	**0.998004**	**0.998004**	**0.998004**	**0.998004**	**0.998004**	1.9515	**0.998004**
	Worst	12.67051	5.928845	**0.998004**	0.998174	2.982105	**0.998004**	9.8069	**0.998004**
	Median	2.982105	**0.998004**	**0.998004**	0.998005	0.998008	**0.998004**	4.9689	**0.998004**
	SD	3.883031	1.093186	**0**	3.3E-05	0.752052	1.51E-16	2.6242	4.12E-17
f_{14}	Mean	-1.03163	-1.03163	-1.03163	-1.03163	-1.03162	-1.03163	**-1.03163**	**-1.03163**
	Best	**-1.03163**	**-1.03163**	**-1.03163**	**-1.03163**	**-1.03163**	**-1.03163**	**-1.03163**	**-1.03163**
	Worst	-1.03163	-1.03163	-1.03163	-1.03163	**-1.03159**	-1.03163	-1.03161	-1.03163
	Median	**-1.03163**	**-1.03163**	**-1.03163**	**-1.03163**	-1.03162	**-1.03163**	**-1.03163**	**-1.03163**
	SD	1.63E-09	**6.78E-16**	**6.78E-16**	1.05E-07	1.09E-05	2.53E-15	**6.78E-16**	4.62E-06
f_{15}	Mean	**0.397887**	**0.397887**	**0.397887**	**0.397887**	0.398243	**0.397887**	**0.397887**	**0.397887**
	Best	**0.397887**	**0.397887**	**0.397887**	**0.397887**	**0.397894**	**0.397887**	**0.397887**	**0.397887**
	Worst	0.397888	0.397887	0.397887	0.397887	**0.39873**	0.397887	**0.397887**	0.397887
	Median	**0.397887**	**0.397887**	**0.397887**	**0.397887**	0.398247	**0.397887**	**0.397887**	**0.397887**
	SD	4.48E-08	**0**	**0**	2.95E-09	0.000237	1.77E-15	**0**	**0**
f_{16}	Mean	3.000002	**3**	**3**	3.000002	3.000006	**3**	**3**	5.7
	Best	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**
	Worst	3.000008	**3**	**3**	3.000007	3.0001	**3**	**3**	30
	Median	3.000002	**3**	**3**	3.000001	3.000001	**3**	**3**	**3**
	SD	2.39E-06	1.38E-15	1.37E-15	1.57E-06	1.81E-05	3.07E-14	**1.32E-15**	8.238472
f_{17}	Mean	-3.86227	**-3.86278**	**-3.86278**	**-3.86278**	-3.85511	**-3.86278**	**-3.86278**	**-3.86278**
	Best	**-3.86278**	**-3.86278**	**-3.86278**	**-3.86278**	**-3.86269**	**-3.86278**	**-3.86278**	**-3.86278**
	Worst	-3.8549	**-3.86278**	**-3.86278**	**-3.86278**	-3.85324	**-3.86278**	**-3.86278**	**-3.86278**
	Median	**-3.86278**	**-3.86278**	**-3.86278**	**-3.86278**	-3.85464	**-3.86278**	**-3.86278**	**-3.86278**
	SD	0.001894	2.7E-15	2.71E-15	7.72E-09	0.002005	8.34E-15	2.71E-15	**2.49E-15**
f_{18}	Mean	-3.26563	-3.21185	-3.23262	**-3.322**	-2.99268	-3.22278	-3.2784	-3.23481
	Best	-3.32199	**-3.322**	**-3.322**	**-3.322**	-3.12904	**-3.322**	**-3.322**	**-3.322**
	Worst	-3.08242	-1.70606	-3.13764	**-3.32199**	-2.05626	-3.20188	-3.2031	-3.2031
	Median	-3.32199	**-3.322**	-3.2031	**-3.322**	-3.0112	-3.20307	**-3.322**	-3.2031
	SD	0.074491	0.293391	0.056085	**1.07E-07**	0.199257	0.045127	0.05827	0.053475
f_{19}	Mean	-9.13958	-5.2304	-9.40617	-10.1064	-2.4069	-8.89066	-8.06397	**-10.1532**
	Best	**-10.1532**	**-10.1532**	**-10.1532**	**-10.1532**	-7.49367	**-10.1532**	**-10.1532**	**-10.1532**
	Worst	-5.0552	-2.63047	-2.68286	-9.21073	-0.49729	-2.68286	-2.68286	**-10.1532**
	Median	-10.1531	-3.89182	**-10.1532**	**-10.1532**	-0.88142	**-10.1532**	**-10.1532**	**-10.1532**
	SD	2.061718	3.176894	2.279414	0.175448	2.243694	2.364052	3.2743	**6.33E-15**
f_{20}	Mean	-10.4028	-4.8571	-10.1803	**-10.4029**	-3.98806	-9.44516	**-10.4029**	**-10.4029**
	Best	**-10.4029**	**-10.4029**	**-10.4029**	**-10.4029**	-8.1251	**-10.4029**	**-10.4029**	**-10.4029**
	Worst	-10.4026	-1.83759	-3.7243	**-10.4029**	-0.90756	-2.7659	**-10.4029**	**-10.4029**
	Median	-10.4028	-4.40599	**-10.4029**	**-10.4029**	-4.61291	**-10.4029**	**-10.4029**	**-10.4029**
	SD	6.97E-05	2.757745	1.219347	9.16E-06	2.348563	2.213355	1.81E-15	**9.9E-16**
f_{21}	Mean	-9.99531	-6.12502	-10.0254	**-10.5364**	-4.34979	-9.92194	-9.7864	**-10.5364**
	Best	**-10.5364**	**-10.5364**	**-10.5364**	**-10.5364**	-8.30572	**-10.5364**	**-10.5364**	**-10.5364**
	Worst	-2.42173	-1.85948	-2.87114	-10.5363	-0.94361	-2.87114	-2.4273	**-10.5364**
	Median	-10.5363	-3.83543	**-10.5364**	**-10.5364**	-4.88751	**-10.5364**	**-10.5364**	**-10.5364**
	SD	2.058728	3.735753	1.944741	2.16E-05	1.890721	1.906547	2.2960	**1.51E-15**

Table 10

The results obtained by iTSA and other considered algorithms on CEC14 test problems.

Function	Test	GWO	PSO	ABC	SCA	SSA	CMA-ES	TSA	TSAFWSS	TSAOBL	iTSA
	Mean	5.261E+07	7.972E+07	1.323E+08	2.899E+08	**8.940E+06**	3.304E+07	3.462E+07	3.505E+07	2.946E+07	2.813E+07
	Best	5.772E+06	7.058E+06	1.797E+07	1.517E+08	1.464E+06	1.522E+07	9.3952E+06	6.659E+06	7.329E+06	7.812E+06
f_1	Worst	1.129E+08	4.093E+08	3.108E+08	5.328E+08	2.012E+07	4.853E+07	7.984E+07	7.211E+07	1.106E+08	6.572E+07
	Median	4.731E+07	3.927E+07	1.400E+08	2.869E+08	8.189E+06	3.296E+07	3.186E+07	3.678E+07	2.630E+07	2.644E+07
	SD	3.101E+07	1.009E+08	7.067E+07	9.960E+07	4.616E+06	8.716E+06	1.775E+07	1.696E+07	2.045E+07	1.089E+07
	Mean	1.209E+09	1.374E+10	6.067E+09	1.929E+10	**1.012E+04**	1.961E+05	1.711E+07	1.984E+07	2.044E+07	1.393E+07
	Best	1.144E+08	1.106E+09	3.744E+08	1.321E+10	2.022E+02	2.376E+04	4.2118E+06	5.663E+06	3.377E+06	3.851E+06
f_2	Worst	5.618E+09	3.979E+10	2.117E+10	2.492E+10	3.433E+04	4.323E+05	3.407E+07	5.225E+07	1.277E+08	3.690E+07
	Median	7.342E+08	9.938E+09	4.208E+09	1.930E+10	8.220E+03	1.706E+05	1.674E+07	1.701E+07	1.681E+07	1.142E+07
	SD	1.189E+09	1.079E+10	5.783E+09	2.864E+09	9.131E+03	1.230E+05	7.731E+06	1.182E+07	2.253E+07	7.723E+06
	Mean	3.422E+04	**6.432E+03**	9.623E+04	4.744E+04	2.663E+04	2.475E+05	1.235E+04	9.258E+03	1.413E+04	6.974E+03
	Best	1.490E+04	5.719E+02	2.422E+04	3.043E+04	1.233E+04	1.284E+05	2.1146E+03	7.592E+02	8.674E+02	7.258E+02
f_3	Worst	5.469E+04	4.181E+04	2.519E+05	6.381E+04	4.677E+04	4.504E+05	5.388E+04	5.419E+04	5.003E+04	4.108E+04
	Median	3.431E+04	2.401E+03	8.887E+04	4.742E+04	2.461E+04	2.398E+05	6.637E+03	4.121E+03	8.288E+03	3.679E+03
	SD	1.097E+04	8.775E+03	5.837E+04	7.878E+03	9.524E+03	6.839E+04	1.371E+04	1.223E+04	1.490E+04	8.952E+03
	Mean	6.358E+02	1.377E+03	1.146E+03	1.649E+03	5.156E+02	**4.201E+02**	5.474E+02	5.530E+02	5.577E+02	5.461E+02
	Best	5.151E+02	4.846E+02	5.208E+02	1.364E+03	4.618E+02	4.193E+02	4.8860E+02	5.182E+02	5.034E+02	4.888E+02
f_4	Worst	1.033E+03	3.316E+03	1.921E+03	2.460E+03	5.841E+02	4.212E+02	5.891E+02	6.218E+02	6.438E+02	6.034E+02
	Median	6.199E+02	1.107E+03	1.158E+03	1.593E+03	5.121E+02	4.200E+02	5.534E+02	5.496E+02	5.578E+02	5.518E+02
	SD	9.356E+01	7.823E+02	3.470E+02	2.372E+02	3.160E+01	5.867E-01	2.648E+01	2.465E+01	3.237E+01	2.631E+01
	Mean	5.210E+02	**5.201E+02**	5.210E+02	5.210E+02	**5.201E+02**	5.209E+02	5.203E+02	5.203E+02	5.202E+02	5.202E+02
	Best	5.208E+02	5.200E+02	5.208E+02	5.207E+02	5.200E+02	5.208E+02	5.2006E+02	5.201E+02	5.200E+02	5.201E+02
f_5	Worst	5.211E+02	5.205E+02	5.210E+02	5.211E+02	5.205E+02	5.210E+02	5.205E+02	5.205E+02	5.204E+02	5.204E+02
	Median	5.210E+02	5.200E+02	5.210E+02	5.210E+02	5.200E+02	5.210E+02	5.203E+02	5.203E+02	5.203E+02	5.202E+02
	SD	6.531E-02	1.197E-01	5.674E-02	8.038E-02	1.221E-01	5.245E-02	1.050E-01	9.533E-02	1.027E-01	1.061E-01
	Mean	6.134E+02	6.236E+02	6.338E+02	6.365E+02	6.191E+02	**6.000E+02**	6.233E+02	6.221E+02	6.234E+02	6.233E+02
	Best	6.089E+02	6.170E+02	6.268E+02	6.319E+02	6.112E+02	6.000E+02	6.1758E+02	6.140E+02	6.159E+02	6.182E+02
f_6	Worst	6.211E+02	6.324E+02	6.390E+02	6.404E+02	6.285E+02	6.000E+02	6.280E+02	6.286E+02	6.276E+02	6.281E+02
	Median	-6.135E+02	6.237E+02	6.346E+02	6.364E+02	6.195E+02	6.000E+02	6.239E+02	6.227E+02	6.234E+02	6.231E+02
	SD	2.850E+00	3.239E+00	3.109E+00	2.121E+00	4.223E+00	0.000E+00	2.797E+00	3.535E+00	2.577E+00	2.645E+00
	Mean	7.209E+02	8.236E+02	7.521E+02	8.718E+02	**7.000E+02**	**7.000E+02**	7.011E+02	7.011E+02	7.012E+02	7.011E+02
	Best	7.027E+02	7.001E+02	7.015E+02	8.078E+02	7.000E+02	7.000E+02	7.0103E+02	7.010E+02	7.010E+02	7.010E+02
f_7	Worst	7.924E+02	9.850E+02	8.117E+02	9.580E+02	7.001E+02	7.000E+02	7.013E+02	7.013E+02	7.014E+02	7.013E+02
	Median	7.164E+02	8.073E+02	7.481E+02	8.637E+02	7.000E+02	7.000E+02	7.011E+02	7.011E+02	7.011E+02	7.011E+02
	SD	1.903E+01	8.662E+01	3.345E+01	3.367E+01	1.484E-02	0.000E+00	9.424E-02	5.805E-02	8.803E-02	5.155E-02
	Mean	-8.795E+02	9.302E+02	8.619E+02	1.057E+03	9.465E+02	**9.488E+02**	8.639E+02	8.596E+02	8.677E+02	8.614E+02
	Best	8.392E+02	8.649E+02	8.336E+02	1.008E+03	8.766E+02	9.092E+02	8.4846E+02	8.415E+02	8.516E+02	8.436E+02
f_8	Worst	9.132E+02	1.008E+03	9.001E+02	1.106E+03	1.006E+03	9.661E+02	8.988E+02	8.828E+02	8.873E+02	8.939E+02
	Median	8.805E+02	9.208E+02	8.622E+02	1.057E+03	9.497E+02	9.523E+02	8.642E+02	8.585E+02	8.675E+02	8.589E+02
	SD	1.500E+01	3.727E+01	1.628E+01	1.844E+01	4.075E+01	1.264E+01	1.196E+01	1.037E+01	1.137E+01	1.361E+01
	Mean	**9.936E+02**	1.079E+03	1.055E+03	1.192E+03	1.041E+03	1.059E+03	1.006E+03	1.016E+03	1.011E+03	9.992E+02
	Best	9.602E+02	1.018E+03	9.658E+02	1.162E+03	9.687E+02	1.038E+03	9.6053E+02	9.738E+02	9.551E+02	9.552E+02
f_9	Worst	1.038E+03	1.171E+03	1.156E+03	1.226E+03	1.118E+03	1.077E+03	1.073E+03	1.060E+03	1.079E+03	1.033E+03
	Median	9.933E+02	1.073E+03	1.060E+03	1.192E+03	1.040E+03	1.061E+03	1.000E+03	1.012E+03	1.013E+03	1.002E+03
	SD	1.979E+01	3.725E+01	4.419E+01	1.783E+01	4.214E+01	8.951E+00	3.118E+01	2.183E+01	2.627E+01	1.894E+01
	Mean	3.232E+03	4.600E+03	5.699E+03	7.275E+03	4.797E+03	8.069E+03	3.189E+03	3.229E+03	**3.178E+03**	3.205E+03
	Best	2.283E+03	3.393E+03	3.997E+03	6.217E+03	3.266E+03	7.329E+03	2.2523E+03	2.682E+03	2.624E+03	2.512E+03
f_{10}	Worst	4.051E+03	5.309E+03	6.990E+03	8.360E+03	6.545E+03	8.721E+03	3.852E+03	3.783E+03	4.037E+03	3.942E+03
	Median	3.236E+03	4.675E+03	5.582E+03	7.312E+03	4.652E+03	8.070E+03	3.156E+03	3.196E+03	3.135E+03	3.200E+03
	SD	4.228E+02	5.295E+02	6.479E+02	5.429E+02	8.601E+02	3.417E+02	3.861E+02	2.993E+02	2.874E+02	3.846E+02

Table 11

The results obtained by iTSA and other considered algorithms on CEC14 test problems.

Function	Test	GWO	PSO	ABC	SCA	SSA	CMA-ES	TSA	TSAFWSS	TSAOBL	iTSA
	Mean	**4.458E+03**	4.806E+03	8.722E+03	8.360E+03	4.887E+03	8.263E+03	5.082E+03	5.091E+03	5.167E+03	5.287E+03
	Best	3.091E+03	2.723E+03	7.509E+03	7.617E+03	3.645E+03	7.491E+03	4.1672E+03	3.922E+03	4.215E+03	4.327E+03
f_{11}	Worst	8.254E+03	6.021E+03	9.406E+03	9.054E+03	6.885E+03	8.778E+03	6.286E+03	6.038E+03	6.458E+03	6.849E+03
	Median	4.068E+03	4.861E+03	8.818E+03	8.335E+03	4.818E+03	8.316E+03	5.016E+03	5.212E+03	5.176E+03	5.181E+03
	SD	1.320E+03	7.364E+02	3.406E+02	3.575E+02	7.462E+02	3.133E+02	4.356E+02	5.705E+02	6.088E+02	5.960E+02
	Mean	1.202E+03	1.201E+03	1.202E+03	1.203E+03	1.201E+03	**1.200E+03**	**1.200E+03**	1.201E+03	**1.200E+03**	1.201E+03
	Best	1.200E+03	1.200E+03	1.202E+03	1.202E+03	1.200E+03	1.200E+03	1.2002E+03	1.200E+03	1.200E+03	1.200E+03
f_{12}	Worst	1.203E+03	1.201E+03	1.203E+03	1.203E+03	1.202E+03	1.200E+03	1.201E+03	1.201E+03	1.201E+03	1.201E+03
	Median	1.202E+03	1.201E+03	1.202E+03	1.203E+03	1.200E+03	1.200E+03	1.200E+03	1.200E+03	1.200E+03	1.200E+03
	SD	1.156E+00	2.214E-01	2.986E-01	2.998E-01	4.883E-01	0.000E+00	2.117E-01	2.423E-01	1.656E-01	2.779E-01
	Mean	**1.300E+03**	1.302E+03	1.302E+03	1.303E+03	1.301E+03	**1.300E+03**	**1.300E+03**	**1.300E+03**	1.301E+03	1.301E+03
	Best	1.300E+03	1.301E+03	1.300E+03	1.303E+03	1.300E+03	1.300E+03	1.3003E+03	1.300E+03	1.300E+03	1.300E+03
f_{13}	Worst	1.301E+03	1.304E+03	1.304E+03	1.304E+03	1.301E+03	1.300E+03	1.301E+03	1.301E+03	1.301E+03	1.301E+03
	Median	1.300E+03	1.303E+03	1.303E+03	1.303E+03	1.301E+03	1.300E+03	1.300E+03	1.300E+03	1.301E+03	1.301E+03
	SD	8.076E-02	1.233E+00	1.222E+00	2.914E-01	1.397E-01	3.405E-02	8.128E-02	1.117E-01	7.228E-02	1.130E-01
	Mean	1.402E+03	1.446E+03	1.421E+03	1.457E+03	**1.400E+03**	**1.400E+03**	**1.400E+03**	**1.400E+03**	**1.400E+03**	1.401E+03
	Best	1.400E+03	1.401E+03	1.400E+03	1.445E+03	1.400E+03	1.400E+03	1.4002E+03	1.400E+03	1.400E+03	1.400E+03
f_{14}	Worst	1.417E+03	1.490E+03	1.459E+03	1.476E+03	1.401E+03	1.401E+03	1.401E+03	1.401E+03	1.401E+03	1.401E+03
	Median	1.400E+03	1.449E+03	1.419E+03	1.457E+03	1.400E+03	1.400E+03	1.400E+03	1.400E+03	1.400E+03	1.400E+03
	SD	3.696E+00	2.531E+01	1.834E+01	8.263E+00	1.810E-01	6.467E-02	2.420E-01	1.732E-01	1.954E-01	2.424E-01
	Mean	1.624E+03	5.289E+03	1.465E+04	8.105E+03	**1.510E+03**	1.514E+03	1.514E+03	1.514E+03	1.513E+03	1.513E+03
	Best	1.506E+03	1.510E+03	1.517E+03	1.780E+03	1.506E+03	1.512E+03	1.5085E+03	1.507E+03	1.508E+03	1.507E+03
f_{15}	Worst	3.237E+03	5.248E+04	3.106E+05	2.166E+04	1.515E+03	1.515E+03	1.520E+03	1.521E+03	1.517E+03	1.520E+03
	Median	1.531E+03	1.685E+03	1.698E+03	6.207E+03	1.510E+03	1.514E+03	1.513E+03	1.513E+03	1.513E+03	1.513E+03
	SD	3.483E+02	1.060E+04	5.645E+04	5.576E+03	2.619E+00	6.375E-01	2.996E+00	3.216E+00	2.321E+00	3.328E+00
	Mean	**1.611E+03**	1.612E+03	1.613E+03	1.613E+03	1.612E+03	1.613E+03	1.612E+03	1.612E+03	1.612E+03	1.612E+03
	Best	1.610E+03	1.611E+03	1.613E+03	1.613E+03	1.610E+03	1.612E+03	1.6108E+03	1.611E+03	1.610E+03	1.611E+03
f_{16}	Worst	1.612E+03	1.613E+03	1.614E+03	1.613E+03	1.613E+03	1.614E+03	1.612E+03	1.612E+03	1.613E+03	1.612E+03
	Median	1.612E+03	1.612E+03	1.613E+03	1.613E+03	1.612E+03	1.613E+03	1.612E+03	1.612E+03	1.612E+03	1.612E+03
	SD	6.930E-01	4.904E-01	2.332E-01	2.286E-01	7.201E-01	2.612E-01	3.826E-01	3.976E-01	5.020E-01	2.849E-01
	Mean	1.374E+06	4.115E+06	4.520E+06	8.132E+06	**5.037E+05**	2.164E+06	6.509E+06	4.708E+06	6.842E+06	5.443E+06
	Best	7.580E+04	2.058E+04	7.401E+05	2.750E+06	8.481E+04	5.059E+05	3.0314E+06	7.562E+05	2.906E+06	1.144E+06
f_{17}	Worst	5.492E+06	3.437E+07	2.415E+07	1.755E+07	1.339E+06	7.247E+06	1.423E+07	1.708E+07	1.408E+07	1.118E+07
	Median	8.666E+05	1.300E+06	3.885E+06	8.070E+06	4.025E+05	1.834E+06	4.893E+06	4.041E+06	5.626E+06	4.971E+06
	SD	1.323E+06	7.360E+06	4.191E+06	3.675E+06	3.114E+05	1.345E+06	3.247E+06	3.115E+06	3.398E+06	2.865E+06
	Mean	9.666E+06	9.031E+07	2.147E+07	2.661E+08	**8.602E+03**	2.012E+06	1.476E+06	2.361E+06	2.166E+06	2.371E+06
	Best	2.515E+03	2.317E+03	9.620E+04	8.287E+07	2.174E+03	4.106E+05	9.9904E+04	5.079E+05	9.727E+04	2.431E+05
f_{18}	Worst	9.300E+07	5.031E+08	1.381E+08	4.887E+08	2.499E+04	5.536E+06	4.705E+06	1.002E+07	6.842E+06	8.144E+06
	Median	1.393E+04	2.572E+04	6.404E+06	2.552E+08	6.805E+03	1.965E+06	1.253E+06	1.690E+06	1.615E+06	1.672E+06
	SD	2.305E+07	1.721E+08	3.134E+07	1.084E+08	6.547E+03	1.003E+06	1.061E+06	2.102E+06	1.700E+06	1.834E+06
	Mean	1.939E+03	1.970E+03	1.946E+03	2.005E+03	1.915E+03	**1.914E+03**	1.935E+03	1.945E+03	1.944E+03	1.924E+03
	Best	1.911E+03	1.910E+03	1.918E+03	1.954E+03	1.909E+03	1.912E+03	1.9111E+03	1.911E+03	1.912E+03	1.912E+03
f_{19}	Worst	1.978E+03	2.126E+03	2.051E+03	2.069E+03	1.921E+03	1.918E+03	1.996E+03	2.012E+03	2.025E+03	1.972E+03
	Median	1.933E+03	1.960E+03	1.928E+03	2.002E+03	1.915E+03	1.913E+03	1.921E+03	1.928E+03	1.925E+03	1.919E+03
	SD	2.044E+01	5.873E+01	3.931E+01	2.837E+01	2.486E+00	1.329E+00	2.918E+01	3.380E+01	3.520E+01	1.642E+01
	Mean	1.798E+04	2.118E+04	7.875E+04	2.229E+04	**9.678E+03**	7.287E+04	1.767E+04	2.458E+04	2.601E+04	1.609E+04
	Best	6.949E+03	4.383E+03	1.805E+04	8.886E+03	3.328E+03	1.710E+04	2.6191E+03	2.566E+03	3.058E+03	3.023E+03
f_{20}	Worst	4.467E+04	9.188E+04	1.793E+05	4.381E+04	2.580E+04	2.193E+05	7.754E+04	7.775E+04	9.967E+04	5.292E+04
	Median	1.574E+04	1.254E+04	6.845E+04	1.966E+04	6.694E+03	6.146E+04	1.146E+04	1.161E+04	1.654E+04	1.336E+04
	SD	8.055E+03	2.091E+04	3.788E+04	8.007E+03	6.543E+03	5.288E+04	1.846E+04	2.336E+04	2.719E+04	1.191E+04

Table 12
The results obtained by iTSA and other considered algorithms on CEC14 test problems.

Function	Test	GWO	PSO	ABC	SCA	SSA	CMA-ES	TSA	TSAFWSS	TSAOBL	iTSA
	Mean	6.645E+05	2.708E+05	9.392E+05	2.109E+06	**1.711E+05**	1.153E+06	1.525E+06	1.382E+06	1.304E+06	1.412E+06
	Best	2.617E+04	1.583E+04	1.098E+05	8.071E+05	2.394E+04	2.874E+05	1.2375E+05	2.916E+05	7.611E+04	1.513E+05
f_{21}	Worst	5.869E+06	9.253E+05	6.726E+06	6.919E+06	6.806E+05	2.640E+06	5.150E+06	5.092E+06	3.475E+06	3.909E+06
	Median	3.662E+05	1.365E+05	7.467E+05	1.763E+06	9.973E+04	1.050E+06	1.176E+06	1.042E+06	9.668E+05	1.127E+06
	SD	1.067E+06	2.630E+05	1.179E+06	1.175E+06	1.707E+05	6.775E+05	1.206E+06	1.124E+06	9.710E+05	1.159E+06
	Mean	**2.595E+03**	2.964E+03	2.762E+03	3.072E+03	2.751E+03	2.777E+03	2.940E+03	2.905E+03	2.951E+03	2.970E+03
	Best	2.262E+03	2.474E+03	2.382E+03	2.757E+03	2.506E+03	2.475E+03	2.6547E+03	2.573E+03	2.571E+03	2.406E+03
f_{22}	Worst	2.896E+03	3.391E+03	3.072E+03	3.391E+03	2.992E+03	3.062E+03	3.263E+03	3.310E+03	3.489E+03	3.323E+03
	Median	2.574E+03	2.914E+03	2.785E+03	3.089E+03	2.728E+03	2.785E+03	2.865E+03	2.868E+03	2.870E+03	3.026E+03
	SD	1.705E+02	2.566E+02	1.823E+02	1.452E+02	1.539E+02	1.445E+02	1.654E+02	1.810E+02	1.855E+02	2.503E+02
	Mean	2.635E+03	2.673E+03	2.665E+03	2.684E+03	2.620E+03	2.627E+03	**2.500E+03**	2.504E+03	**2.500E+03**	**2.500E+03**
	Best	2.623E+03	2.619E+03	2.615E+03	2.656E+03	2.616E+03	2.615E+03	2.5000E+03	2.500E+03	2.500E+03	2.500E+03
f_{23}	Worst	2.680E+03	2.798E+03	2.803E+03	2.747E+03	2.634E+03	2.632E+03	2.500E+03	2.621E+03	2.500E+03	2.500E+03
	Median	2.633E+03	2.663E+03	2.649E+03	2.681E+03	2.618E+03	2.629E+03	2.500E+03	2.500E+03	2.500E+03	2.500E+03
	SD	1.019E+01	4.415E+01	4.459E+01	2.016E+01	5.001E+00	5.444E+00	8.790E-10	2.216E+01	4.751E-09	8.444E-14
	Mean	**2.600E+03**	2.668E+03	2.664E+03	2.601E+03	2.639E+03	2.646E+03	**2.600E+03**	**2.600E+03**	**2.600E+03**	**2.600E+03**
	Best	2.600E+03	2.641E+03	2.637E+03	2.600E+03	2.628E+03	2.637E+03	2.6000E+03	2.600E+03	2.600E+03	2.600E+03
f_{24}	Worst	2.600E+03	2.723E+03	2.697E+03	2.626E+03	2.658E+03	2.649E+03	2.600E+03	2.600E+03	2.600E+03	2.600E+03
	Median	2.600E+03	2.665E+03	2.660E+03	2.600E+03	2.640E+03	2.648E+03	2.600E+03	2.600E+03	2.600E+03	2.600E+03
	SD	2.825E-03	2.001E+01	1.515E+01	4.620E+00	9.175E+00	3.148E+00	3.387E-02	4.900E-04	3.441E-04	6.569E-06
	Mean	2.711E+03	2.715E+03	2.711E+03	2.730E+03	2.714E+03	2.708E+03	**2.700E+03**	**2.700E+03**	**2.700E+03**	**2.700E+03**
	Best	2.700E+03	2.705E+03	2.705E+03	2.702E+03	2.700E+03	2.703E+03	2.7000E+03	2.700E+03	2.700E+03	2.700E+03
f_{25}	Worst	2.717E+03	2.739E+03	2.728E+03	2.749E+03	2.718E+03	2.715E+03	2.700E+03	2.700E+03	2.700E+03	2.700E+03
	Median	2.712E+03	2.712E+03	2.710E+03	2.731E+03	2.714E+03	2.708E+03	2.700E+03	2.700E+03	2.700E+03	2.700E+03
	SD	3.768E+00	9.021E+00	6.412E+00	9.273E+00	3.273E+00	2.812E+00	8.402E-13	8.487E-13	3.681E-13	1.194E-13
	Mean	2.747E+03	2.738E+03	2.702E+03	2.703E+03	2.701E+03	2.720E+03	**2.700E+03**	**2.700E+03**	**2.700E+03**	**2.700E+03**
	Best	2.700E+03	2.701E+03	2.700E+03	2.702E+03	2.700E+03	2.700E+03	2.7003E+03	2.700E+03	2.700E+03	2.700E+03
f_{26}	Worst	2.801E+03	2.977E+03	2.704E+03	2.704E+03	2.701E+03	2.900E+03	2.701E+03	2.701E+03	2.701E+03	2.701E+03
	Median	2.701E+03	2.704E+03	2.702E+03	2.703E+03	2.701E+03	2.700E+03	2.700E+03	2.700E+03	2.700E+03	2.700E+03
	SD	5.053E+01	7.317E+01	1.237E+00	4.459E-01	1.410E-01	6.095E+01	8.422E-02	7.980E-02	1.092E-01	8.156E-02
	Mean	3.342E+03	3.681E+03	3.866E+03	3.698E+03	3.509E+03	3.521E+03	**2.917E+03**	2.980E+03	2.937E+03	2.960E+03
	Best	3.109E+03	3.110E+03	3.692E+03	3.182E+03	3.103E+03	3.000E+03	2.9000E+03	2.900E+03	2.900E+03	2.900E+03
f_{27}	Worst	3.531E+03	3.957E+03	4.032E+03	4.010E+03	3.722E+03	4.028E+03	3.168E+03	3.806E+03	3.784E+03	3.555E+03
	Median	3.359E+03	3.776E+03	3.880E+03	3.869E+03	3.486E+03	3.725E+03	2.900E+03	2.900E+03	2.900E+03	2.900E+03
	SD	1.264E+02	2.527E+02	8.663E+01	3.212E+02	1.314E+02	4.696E+02	6.322E+01	2.458E+02	1.650E+02	1.730E+02
	Mean	3.961E+03	4.807E+03	3.321E+03	5.000E+03	3.866E+03	3.223E+03	3.026E+03	**3.027E+03**	3.028E+03	3.055E+03
	Best	3.676E+03	4.331E+03	3.216E+03	4.339E+03	3.685E+03	3.208E+03	3.0000E+03	3.000E+03	3.000E+03	3.000E+03
f_{28}	Worst	4.612E+03	5.954E+03	3.604E+03	6.170E+03	4.392E+03	3.320E+03	3.780E+03	3.811E+03	3.832E+03	3.905E+03
	Median	3.867E+03	4.749E+03	3.299E+03	4.921E+03	3.829E+03	3.218E+03	3.000E+03	3.000E+03	3.000E+03	3.000E+03
	SD	2.367E+02	3.793E+02	8.331E+01	3.942E+02	1.634E+02	2.240E+01	1.423E+02	1.481E+02	1.519E+02	2.104E+02
	Mean	3.673E+05	7.185E+06	3.207E+03	1.719E+07	4.925E+06	**3.128E+03**	5.695E+06	5.467E+06	5.317E+06	6.710E+06
	Best	8.649E+03	1.384E+04	3.122E+03	7.739E+06	1.023E+04	3.123E+03	6.2180E+03	6.207E+03	5.122E+03	5.262E+03
f_{29}	Worst	2.825E+06	1.913E+07	3.788E+03	3.623E+07	2.421E+07	3.136E+03	1.624E+07	1.112E+07	1.259E+07	2.635E+07
	Median	5.221E+04	7.635E+06	3.149E+03	1.367E+07	2.787E+04	3.128E+03	8.405E+06	8.405E+06	8.536E+06	8.667E+06
	SD	6.930E+05	6.017E+06	1.500E+02	8.633E+06	7.466E+06	3.024E+00	5.664E+06	4.510E+06	4.893E+06	6.542E+06
	Mean	5.984E+04	1.031E+05	**3.710E+03**	3.539E+05	2.044E+04	4.593E+03	1.791E+04	2.104E+04	1.774E+04	1.658E+04
	Best	1.584E+04	4.467E+03	3.343E+03	9.863E+04	1.001E+04	3.877E+03	9.3813E+03	8.085E+03	8.186E+03	7.409E+03
f_{30}	Worst	1.545E+05	5.171E+05	4.106E+03	5.767E+05	4.917E+04	5.187E+03	3.153E+04	4.293E+04	4.197E+04	2.737E+04
	Median	4.530E+04	5.403E+04	3.769E+03	3.281E+05	1.695E+04	4.573E+03	1.744E+04	1.819E+04	1.654E+04	1.608E+04
	SD	3.864E+04	1.178E+05	2.005E+02	1.321E+05	9.809E+03	3.201E+02	6.125E+03	9.410E+03	7.354E+03	5.235E+03

Table 13
Comparison of iTSA with other algorithms by Wilcoxon signed-rank tests on unimodal and multimodal classical test problems ($f_1 - f_{12}$).

	TSA	**GWO**	**PSO**	**DE**	**ABC**	**SCA**	**SSA**	**CMA-ES**
p	9.7656E-04	9.7656E-04	7.3242E-04	0.0161	0.0215	2.4414E-04	2.4414E-04	0.0645
h	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**0**
rank	+	+	+	+	+	+	+	+

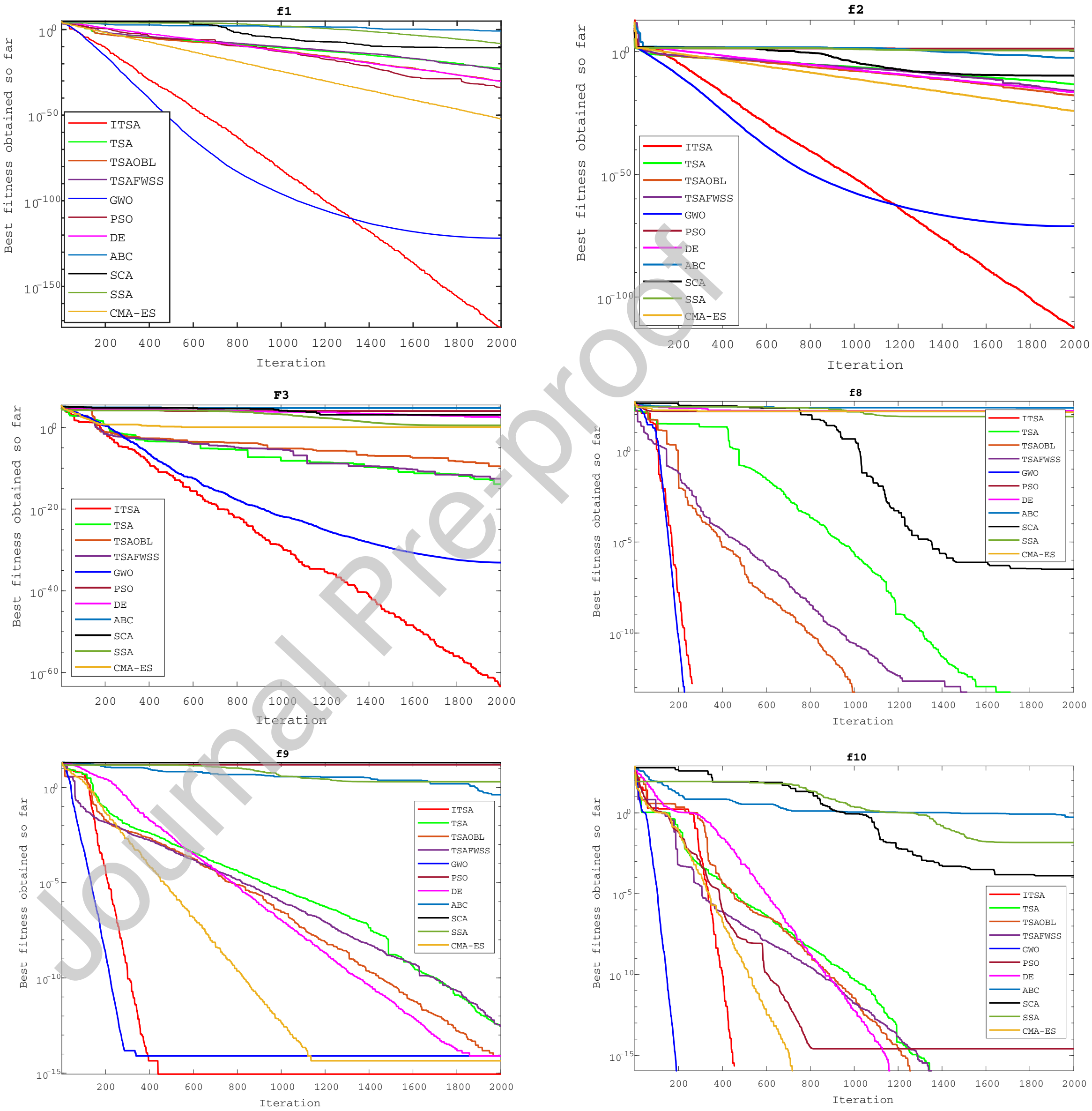

Figure 2: The convergence curves of iTSA with other state of the art algorithms.

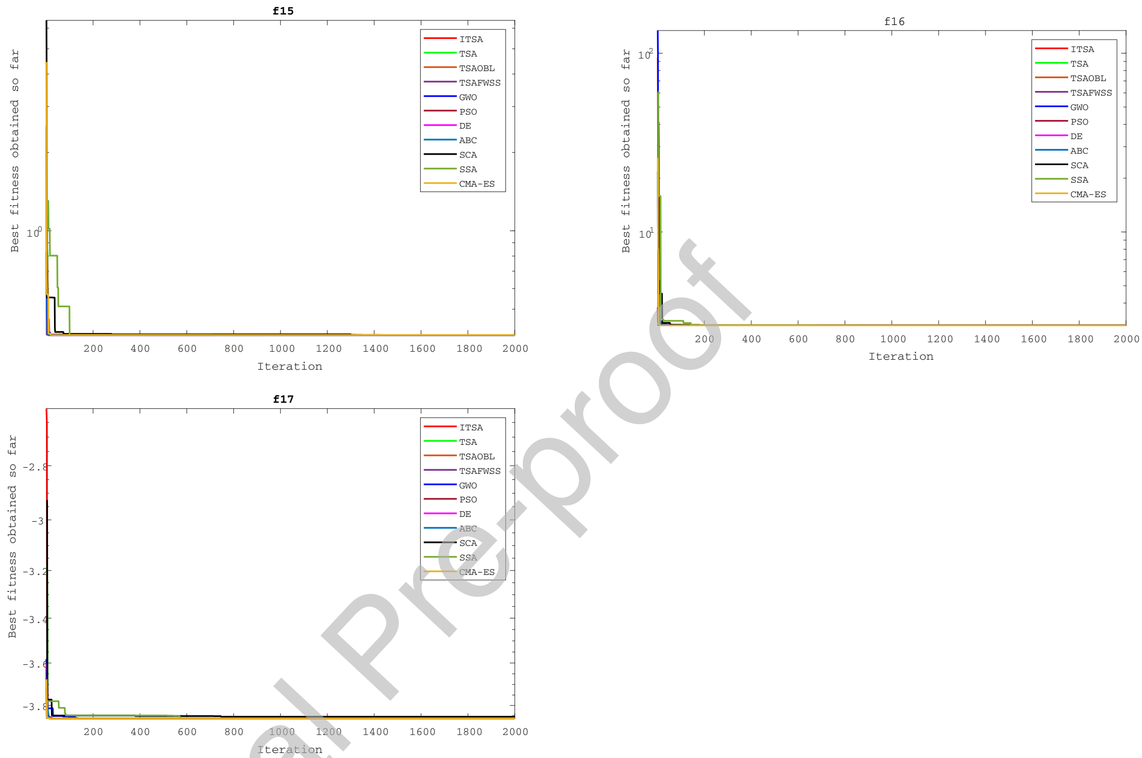

Figure 3: The convergence curves of iTSA with other state of the art algorithms.

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Table 14
Comparison of iTSA with other algorithms by Wilcoxon signed-rank tests on fixed dimensional multimodal classical problems ($f_{13} - f_{21}$).

	TSA	**GWO**	**PSO**	**DE**	**ABC**	**SCA**	**SSA**	**CMA-ES**
p	1	0.0645	0.5000	0.5000	0.0195	0.0020	0.2969	1
h	**0**	**0**	**0**	**0**	**1**	**1**	**0**	**0**
rank	+	+	+	−	+	+	+	-

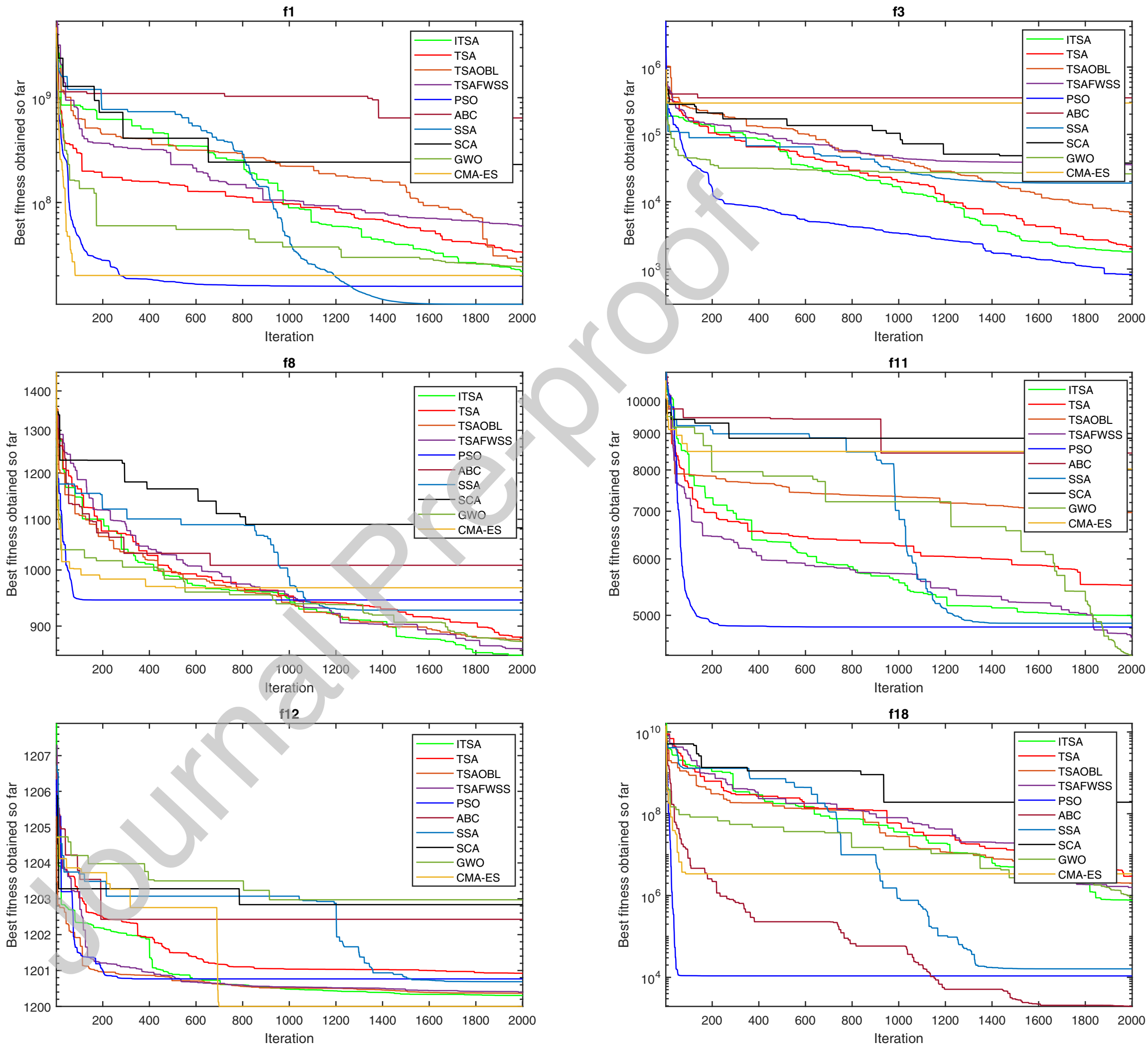

Figure 4: The convergence curves of iTSA with other state of the art algorithms.

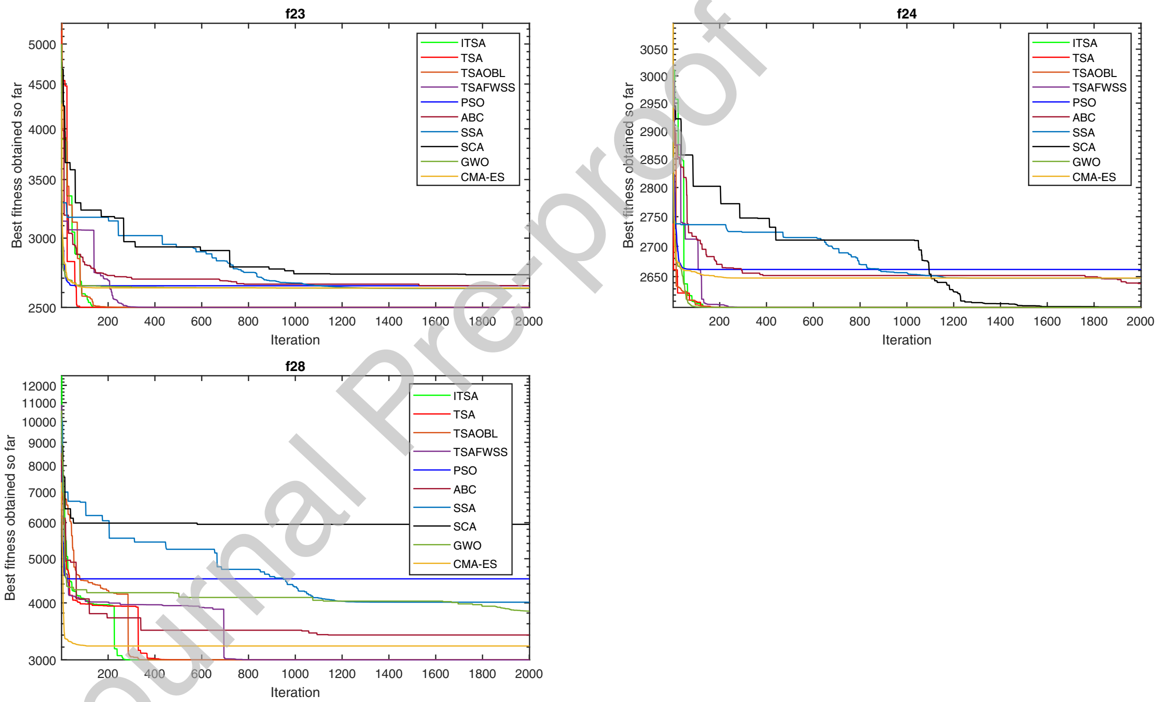

Figure 5: The convergence curves of iTSA with other state of the art algorithms.

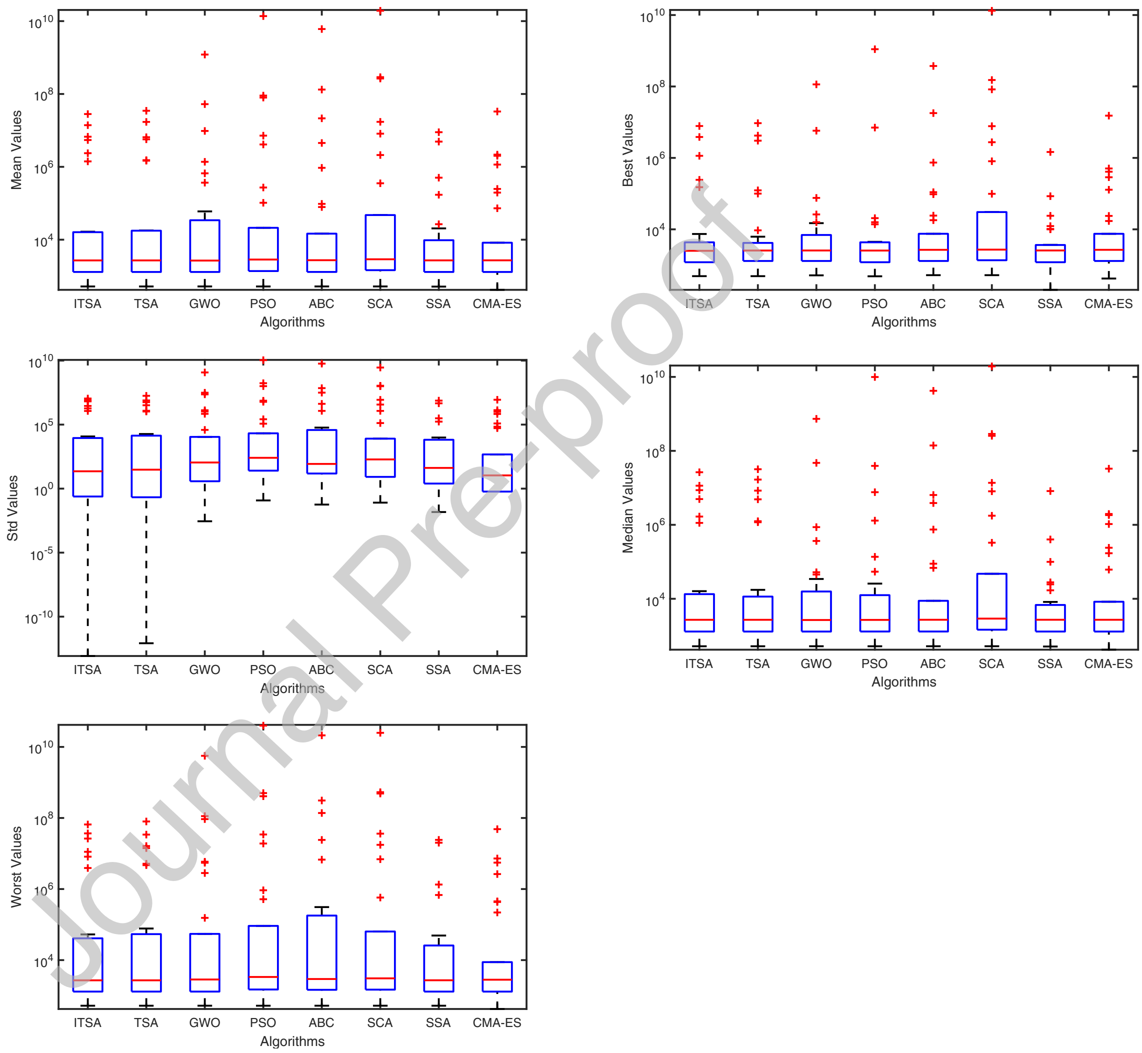

Figure 6: Box plots for the results of the considered algorithms on CEC14 test problems.