Mutation-driven Grey Wolf Optimizer with Modified Search Mechanism

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Abstract

The Grey wolf optimizer (GWO) is a recently introduced popular swarm-intelligence-based metaheuristic algorithm, compared to other algorithms, it has shown competitive performance. Despite its popularity, the conventional GWO suffers from slow convergence rate and tendency to stuck in local optima. Therefore, there is a chance of improvement in the search mechanism of the GWO through different operators. To improve the performance of the GWO, this paper proposes a new variant of the GWO called Mutation-driven Modified Grey wolf optimizer and denoted by MDM-GWO. The MDM-GWO combines a new update search mechanism, modified control parameter, mutation-driven scheme, and greedy approach of selection in the search procedure of the GWO. The performance of the proposed MDM-GWO is evaluated on 23 well-known standard benchmark problems of wide varieties of complexities and four real-world engineering design problems. The numerical results, statistical tests, convergence, and diversity curves, and comparisons among several algorithms show the superiority of the proposed MDM-GWO.

Keywords: Grey wolf optimizer; Exploration and Exploitation; Mutation; Swarm Intelligence.

1. Introduction

Over the past decade, various metaheuristic algorithms have been developed and applied in several fields of science and engineering applications

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to test their ability to find the optimal solution. In the field of different applications such as machine learning, signal processing, neural networks, industry, medical applications, chemistry and artificial intelligence (Tharwat, 2019; Russell and Norvig, 2002; Li et al., 2015; Anitha et al., 2007; Tharwat et al., 2017; Liu et al., 2011). These problems cannot be solved by traditional optimization methods because of discontinuity, non-differentiability, convexity, high dimensions and complicated interaction among variables. Some traditional optimization methods are random walk method, steepest descent method, conjugate gradient method, quasi-Newton method, and others. In order to takle these types of problems, many metaheuristics are developed. Simple framework, easy implementation, and derivative-free mechanism are the special features of these methods that have attracted great research interests and have been widely applied to practical problems. In the literature, metaheuristic algorithms have the capacity to solve several real-life applications (Abbassi et al., 2020; Ridha et al., 2020; Peraza et al., 2020; Carreon et al., 2020).

Some of the well-known meta-heuristic algorithms are Genetic algorithm (GA) (Holland, 1975), Particle swarm optimization (PSO) (Kennedy and Eberhart, 1995), Ant colony optimization (ACO) algorithm (Dorigo and Stützle, 2019), Artificial bee colony (ABC) algorithm (Karaboga and Basturk, 2007), Gravitational search algorithm (GSA) (Rashedi et al., 2009), Spider monkey optimization (SMO) (Bansal et al., 2014), Biogeography-based optimization (BBO) (Simon, 2008), Teaching-learning based optimization (TLBO) (Rao et al., 2011) and many more. Some of the other algorithms which are recently developed such as Grey wolf optimizer (GWO) (Mirjalili et al., 2014), Harris hawks optimizer (HHO) (Heidari et al., 2019), Sine cosine algorithm (SCA) (Mirjalili, 2016), Arithmetic optimization algorithm (AOA) (Abualigah et al., 2021), Golden eagle optimizer (GEO) (Mohammadi-Balani et al., 2021), etc.

GWO is a recently developed meta-heuristic algorithm (Mirjalili et al., 2014). It is inspired by the leadership and hunting behavior of the grey wolves in nature. The aim of adopting GWO for the study is its different search mechanism based on the leadership behavior of grey wolves. In GWO, throughout the iterations, the search direction of the GWO is decided by the leading wolves (alpha, beta, and delta), which helps to ensure a fast convergence speed. Hence, the multiple solution-based guided search scheme of GWO provides an exploration and exploitation towards elite and promising areas of the search space so that a better balance between exploitation and exploration can be established. Although it demonstrates its capacity of balancing exploitation and exploration, in some complex optimization problems, it experiences improper balance between exploitation and exploration and towards local optima during the search procedure. In the last few years, GWO has become quite popular. In the literature, significant growth in the application of GWO is observed for solving different real-life application problems such as economic dispatch problem (Jayakumar et al., 2016; Kamboj et al., 2016), feature selection (Emary et al., 2016), parameter estimation in surface waves (Song et al., 2015), scheduling problem (Komaki and Kayvanfar, 2015), training of q-Gaussian radial basis (Muangkote et al., 2014), wind speed forecasting (Song et al., 2018), power dispatch problem (Sulaiman et al., 2015), and many others. Although the application of GWO demonstrates its sufficient ability in terms of exploration and exploitation, but still in some cases, it suffers the issue of stagnation at local optima and inappropriate balance between exploration and exploitation. Therefore, in the literature, several attempts have been made to improve the search mechanism of GWO. Mittal et al. (2016) proposed a modified version of the GWO by adopting a non-linear transition control parameter strategy. This modification aimed to achieve an appropriate balance between exploration and exploitation. Experimental results in the paper illustrate that the proposed strategy has improved the search performance of the original GWO but is still unable to provide near optimal solutions for multimodal problems. In order to enhance the exploration skills of the GWO, Long et al. (2018) proposed exploration-enhanced GWO (EEGWO). In this algorithm, a new modified position update equation is used to explore more areas of the search space. In addition to this, a non-linear control parameter strategy is also embedded in the EEGWO to balance the diversity and convergence speed. Yu et al. (2021) proposed OGWO that combines the GWO with OBL and nonlinear control parameter to improve the performance of the original GWO. This variant has performed well on the considered benchmark problem, but it suffers from the problem of skipping true solutions during the search process on multimodal problems. A beetle antenna strategy in the GWO algorithm is integrated by Fan et al. (2021a) to enhance exploration capability and to reduce unnecessary searches. This variant cannot increase the exploitation skills, and therefore, the results on unimodal problems are not good enough. Another improved GWO algorithm, named LGWO, was proposed by Heidari and Pahlavani (2017). They have integrated greedy selection and Levy flight strategies with a modified hunting phase to alleviate the stagnation problem in GWO. In order to increase the convergence rate, exploration, and exploitation abilities, Bansal and Singh (2020) proposed IGWO. This improvement is conducted using the explorative equation and opposition-based learning (OBL). These strategies are effective in cases where the optima are far from the current state of solution. A group-based synchronous–asynchronous GWO is developed by Rodríguez et al. (2021), where asynchronous–asynchronous processing scheme is incorporated to increase the diversity of the GWO population. Dhargupta et al. (2020) used Spearman's correlation coefficient to determine the position of omega wolves. Moreover, in this proposed method, OBL is combined with GWO, but instead of opposing all the dimensions of the wolves, only a few of them are obtained using OBL. This helps in avoiding unnecessary exploration and achieving fast convergence. Gupta and Deep (2020) proposed mGWO by modifying the GWO search mechanism based on the personal best history of wolves, crossover, and greedy selection. These strategies have improved the global exploration and local exploitation abilities of the original GWO. Furthermore, in order to strike a good balance between exploration and exploitation, GWO is hybridized with other metaheuristics. For example, Gaidhane and Nigam (2018) proposed a hybrid version of GWO and ABC called GWO-ABC. In this algorithm, the information sharing strategy of employed bee from ABC is integrated with the leadership behavior of the GWO to boost exploration ability. In (Singh and Singh, 2017), search strategies of GWO and SCA are combined, and a new hybrid method called GWO-SCA is proposed. The aim of this hybridization is to utilize the exploitation nature of the GWO and the explorative nature of the SCA. By adding the principle of survival of the fittest (SOF), biological evolution, and differential evolution algorithm, a new hybrid method called IGWO is proposed to avoid the situation of falling into local optima and to accelerates the convergence speed of the GWO (Wang and Li, 2019).

In this study, four different strategies are explored to enhance the performance of the conventional GWO. These components are the modified search mechanism of the GWO, modified control parameter, mutation-driven search scheme, and greedy approach. The modified search mechanism and mutation-driven search scheme increase the exploration strength and enhance the capability to jump out from local optima, respectively. In the mutation-driven scheme, Levy flight strategy distributed random numbers are used due to the specific nature of this distribution, which infrequently generates large numbers and helps avoid the prone towards local optima. The modified control parameter, which is used for transition from exploration to exploitation in GWO, is modified to adopt the non-linear nature of search and increase exploitation capability. Meanwhile, to avoid high exploration during the search procedure, a greedy search approach is also used between the newly obtained position and previous position. All these strategies are merged in the GWO, and the paper proposes a new algorithm called MDM-GWO.

The main structure of this paper is as follows: The introduction and literature of the GWO is summarized in Section 1. Briefly describe the theory of conventional GWO and steps of the algorithm in Section 2. The major contribution i.e. our proposed method is discussed in Section 3. Simulation results of the proposed algorithm and four engineering design problems are presented in Section 4. Finally, Section 5 concludes this paper and suggests some future ideas.

2. Conventional Grey Wolf Optimizer

GWO is a new meta-heuristic optimization algorithm introduced in 2014 by Mirjalili et al. (Mirjalili et al., 2014). It is based on the leadership level and hunting activities of wolves in nature. In nature, Grey wolves always prefer to live in a group. In this group, the number of grey wolves is 5-12 on average. Wolves are divided into four categories, namely- alpha, beta, delta, and omega wolves. First is the alpha wolf which is called the dominant wolf in the group, and all the decisions in the group are taken by him/her. The secondary wolf is the beta wolf which plays the role of the advisor in the absence of the alpha wolf. The third wolf is the delta wolf which is the caretaker of the group. Fourth, wolves (remaining wolves) are omega wolves that are allowed to eat in the end. Like other swarm-intelligence-based metaheuristic algorithms, GWO initializes the population. Then, the wolves update their position in the solution space around the prey.

2.1. Mathematical model

In this section, we described the leadership behavior, encircling, and hunting behavior mathematically in the following manner:

2.1.1. leadership behavior

In this subsection, the top three fittest solutions are assumed as leader wolves, alpha (α), beta (β), and delta (δ), and remaining wolves are omega

(ω) wolves. These follower wolves (ω wolves) update their states by the guidance of leading wolves (α, β, and δ).

2.1.2. Encircling behavior

The encircling behavior can be modelled in the following mathematical way:

$$D = |C \times X_{l.w}^t - X_w^t| \tag{1}$$

$$X_w^{(t+1)} = X_{l.w}^t - A \times D \tag{2}$$

where t indicates the current iteration number. $X_{l.w}^t$ is the position of leading wolves (α, β and δ) at t^{th} iteration and $X_w^{(t+1)}$ is the position of grey wolves in next iteration. The coefficient vectors A and C are defined in equations (3) and (4). D is the difference vector between the grey wolf and the leader wolves.

$$A = 2 \times a \times rand_1 - a \tag{3}$$

$$C = 2 \times rand_2 \tag{4}$$

$rand_1$ and $rand_2$ are uniformly random numbers in the interval $[0, 1]$. a is linearly decremented from 2 to 0 with the number of iterations and defined in equation (5).

$$a = 2 - 2\left(\frac{t}{Maxiter}\right) \tag{5}$$

where, $Maxiter$ is the maximum iterations number.

2.1.3. Hunting behavior

During the hunting process, it is considered that all the leading wolves have better knowledge about the prey's location. In this manner, each wolf updated their positions based on the positions of leading wolves using the following equations are:

$$D_\alpha = |C_1 \times X_{\alpha.w}^t - X_w^t| \tag{6}$$

$$D_\beta = |C_2 \times X_{\beta.w}^t - X_w^t| \tag{7}$$

$$D_\delta = |C_3 \times X^t_{\delta.w} - X^t_w| \tag{8}$$

where, $X^t_{\alpha.w}$, $X^t_{\beta.w}$ and $X^t_{\delta.w}$ are the position of α, β and δ wolves at t^{th} iteration. C_1, C_2 and C_3 are coefficient vectors as defined in equation (4). After obtaining the difference vectors D_α, D_β and D_δ, the new position of the grey wolf at $(t+1)^{th}$ is calculated as follows:

$$X^{(t+1)}_w = \frac{X^t_{w1} + X^t_{w2} + X^t_{w3}}{3} \tag{9}$$

where,

$$X^t_{w1} = X^t_{\alpha.w} - A_1 \times D_\alpha \tag{10}$$

$$X^t_{w2} = X^t_{\beta.w} - A_2 \times D_\beta \tag{11}$$

$$X^t_{w3} = X^t_{\delta.w} - A_3 \times D_\delta \tag{12}$$

the coefficient vectors A_1, A_2 and A_3 are defined in equation (3).
The exploration and exploitation behavior balance is determined by random and adaptive vectors A and C. If $|A| > 1$ and $C > 1$, the grey wolves population expands its search scope and turns towards exploration. The wolf exploits the search space when $|A| < 1$ and $C < 1$. Therefore, the parameters A and C play key roles in the GWO algorithm. Algorithm 1 shows pseudo code of the conventional GWO.

3. Proposed Mutation-driven Grey Wolf Optimizer with Modified Search Mechanism

The conventional GWO converges quickly into local optima. However, this fast convergence comes at the cost, chances of getting stuck to the local optima when dealing with complex optimization problems or when the dimension of the problem increases. The reason for this degraded performance is the low exploration capability of the conventional GWO (Mirjalili et al., 2014). Hence, increasing the exploration capability of the algorithm may provide a better version of the conventional GWO. However, increasing the exploration capability directly affects the exploitation capability of the conventional GWO, which may result an increased error in the obtained solution. Thus, to improve the overall performance of

Algorithm 1 Grey Wolf Optimizer (GWO) algorithm

```
Initialize the parameters
Initialize the grey wolves positions say X_w(w = 1, 2, ... N)
Evaluate the fitness say f(X_w) at X_w
Select α, β, and δ wolves
initialize a, A and C
initialize t = 0
while Termination criteria is meet do
  for for each wolf do
    update the position of wolves using equation (9)
  end for
  update a, A and C
  update α, β, and δ wolves
  t=t+1
end while
Return α wolf
```

the conventional GWO, it is necessary that the work not only exploration capability but also over exploitation capability of the conventional GWO. Therefore, in this paper, we have combined four different strategies: modified search mechanism, modified non-linear control parameter, mutation-driven search scheme, and the greedy selection approach. Altogether, these strategies are expected to enhance the exploration ability of the algorithm while maintaining the exploration capability of the conventional GWO. The description of each embedded strategy is as follows:

3.1. Modified Search Mechanism

The guidance provided by α, β, and δ wolves is an important factor when updating the position of ω wolves. In the conventional GWO, the average of the estimated positions of the three best search agents (α, β, and δ) are considered to obtain the new position of the wolves. Logically, the conventional GWO provides a new position at the centroid of a convex region surrounded by the resulting direction points obtained using α, β, and δ wolves. Hence the new position of a wolf lies in the neighborhood of positions which are obtained in the direction of the estimated positions of the α, β, and δ wolves. This logic does not work efficiently when these leading wolves are either stuck in local optima or very far from the optima. Hence, instead of taking the centroid position of a convex region, we have chosen an affine combination of the estimated positions of the three best search agents (α, β, and δ) with coefficient value in the range $(-1, 3)$ (explained below). For exploring the large area around the positions obtained in the direction of α, β, and δ wolves and to decrease the possibility of stucking into local optima. Mathematically, this new search mechanism can be expressed by:

$$Y_w^{(t+1)} = R_1 \times X_{w1}^t + R_2 \times X_{w2}^t + R_3 \times X_{w3}^t \tag{13}$$

where, $Y_w^{(t+1)}$ is the position of grey wolf in the $(t+1)^{th}$ iteration and X_{w1}^t, X_{w2}^t and X_{w3}^t are obtained by equations (10), (11) and (12) in the t^{th} iteration, respectively. Initially, R_1, R_2 and R_3 are uniformly distributed random numbers in the interval $(0, 1)$ and they are recalculated further as follows:

$$R_k = 4 \times \frac{R_k}{\sum_{i=1}^{3} R_i} - 1, \quad k = 1, 2, 3. \tag{14}$$

Equation (14) is inspired by a novel multi-parent crossover operator which is proposed by Zeng et al (Zeng et al., 2007). This new update search mechanism randomly contributes the effects of α, β and δ guidance in the search procedure and therefore useful in enhancing the capability of exploration.

3.2. Modified Control Parameter

As discussed in the literature shows, the conventional GWO suggests a balance between exploitation and exploration through the control parameter a, which is linear in iteration counter t (Mirjalili et al., 2014; Heidari and Pahlavani, 2017; Ibrahim et al., 2018; Zhang et al., 2018). Therefore, a modification in this parameter may vary the exploitation and exploration capabilities of the algorithm. Also, to mimic the non-linear search procedure of the GWO, this control parameter can be transformed to a non-linear function (Long et al., 2019), which is given by:

$$a = (a_{start} - a_{end}) \times E + a_{end} \tag{15}$$

where,

$$E = exp\Big(- \frac{t^2}{(K \times Maxiter)^2} \Big)$$

and t indicates the current iteration, $Maxiter$ indicates the maximum number of iterations, K is the non-linear modulation index, and a_{start} and a_{end} are the initial and final values of parameter a, respectively. In order to accomplish our goal of achieving better exploitation, we have fixed the value of K to 0.3, and the value of a_{start} and a_{end} are 2 and 0, respectively. The comparison between the linear and the proposed non-linear control parameter is demonstrated in Figure 1. In Figure 1, the linear behavior of the control parameter indicates 50% global exploration and 50% local exploitation, and the proposed non-linear behavior of the control parameter indicates 25% global exploration and 75% local exploitation. Therefore, the utilization of the non-linear control parameter places more emphasis on exploitation, thereby enhancing the local search ability and reducing the chance to skip true solutions during the search procedure. This is required as a modified search mechanism increases the exploration is going to affect the exploitation

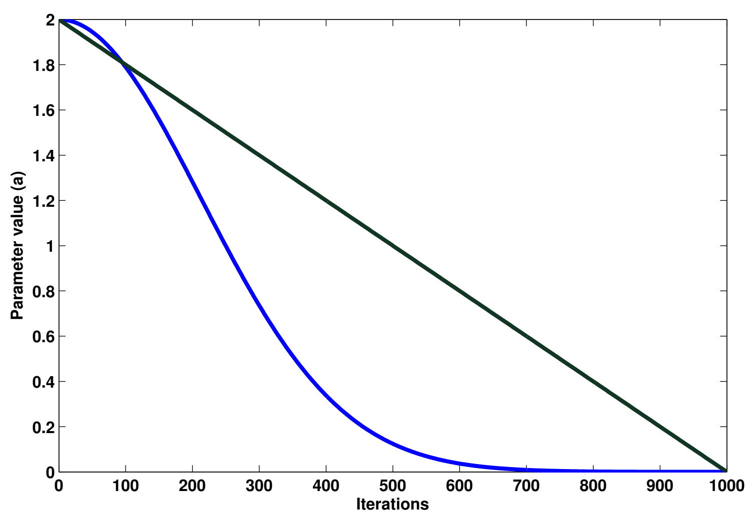

Figure 1: Linearly and non-linearly decreasing curve for parameter a

3.3. Mutation-driven Search Scheme

In this scheme, we adopted the Levy-flight based mutation scheme to enhance the global search ability. This mutation allows mutating the wolves to avoid the chance of falling into local optima and provides a move to jump out from that local optima. In our proposed algorithm, this mutation scheme is applied based on a mutation probability p_m. A larger probability allows a high chance to mutate the wolves, and that sometimes harmful and skips the true solutions. Therefore, we have fixed it to 0.1 to keep the effect of the modified search mechanism. The following equation can express the proposed mutation scheme.

$$Z_w^{t+1} = X_w^t + E \times L(\theta) \tag{16}$$

where, E is a scalar value that controls the mutation step size which is explained in the above formula.

In equation (16), $L(\theta)$ is a Levy-flight distributed random number. Levy-flight is a random walk in which the step lengths determine the steps, and the jumps conform to a Levy distribution. In this paper, for generating step lengths, Mantegna algorithm (Leccardi, 2005; Soneji and Sanghvi, 2012) is used. Mantegna algorithm performs several small steps and occasionally a big step which helps avoid the chance of falling into local optima and provides a move to jump out from that local optima. Using the Mantegna algorithm, 50 step sizes have been drawn to form a consecutive 50 steps of 2D Lévy flight distribution, shown in Figure 2. Mathematically, steps are calculated as follows:

$$L(\theta) = \frac{u}{|v|^{\frac{1}{\theta}}} \tag{17}$$

where u and v parameters in the above equation have normal distributions and are obtained as follows:

$u \sim N(0, \sigma_u^2), \quad v \sim N(0, \sigma_v^2),$

σ_u and σ_v are defined by the following formula:

$\sigma_u = \left(\frac{\Gamma(1+\theta)\cdot\sin(\pi\theta/2)}{\Gamma[(1+\theta)/2]\cdot\theta\cdot 2^{(\theta-1)/2}}\right)^{1/\theta} \qquad \sigma_v = 1.$

For $|s| \geq |s_0|$, this distribution (for s) obeys the expected Levy distribution. s_0 is the smallest step length (Yang and Deb, 2010) and $\Gamma(.)$ is the Gamma function and calculated as follows:

$$\Gamma(1+\theta) = \int_0^\infty t^\theta e^{-t} dt \tag{18}$$

In a special case, when θ is an integer, then we have $\Gamma(1+\theta) = \theta!$. In this work, the value of θ is fixed as 1.

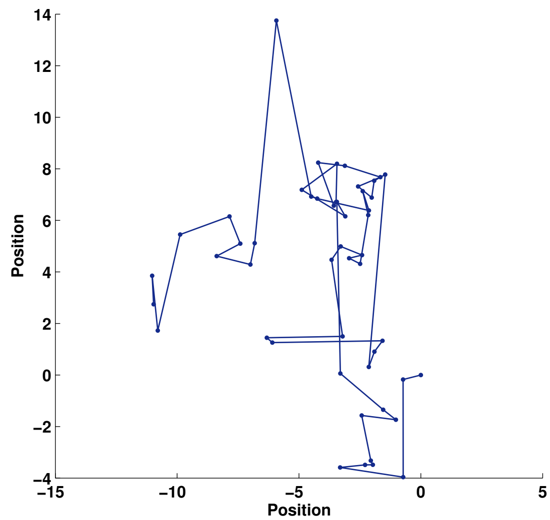

Figure 2: A sequence of 50 consecutive steps of Levy flights

3.4. Greedy Selection Approach

The greedy selection approach is used to decide whether the newly updated individual is to be accepted in the subsequent stage of evolution or not. It is an approach of picking one best solution between two available solutions based on their fitness values. This approach is used to perform the search with better solutions instead of poor ones. This greedy nature helps increase the algorithm's exploitation capability and perform the search in the elite direction obtained so far. In our proposed MDM-GWO, this greedy selection is applied at two different places, one is when the positions are obtained from the modified search mechanism, and another is when the mutation-driven scheme is applied. For a minimization problem, one way of selecting one solution between two solutions X and Y, using greedy selection approach, can be defined as follows:

$$Z = \begin{cases} X & \text{if } f(X) \leq f(Y) \\ Y & \text{if } f(X) > f(Y) \end{cases} \tag{19}$$

All the above described strategies are embedded into the conventional GWO to proposed a new variant of GWO called MDM-GWO. The pseudocode for MDM-GWO with all of the proposed strategies is presented in Algorithm 2.

Algorithm 2 Mutation-driven Grey Wolf Optimizer with Modified Search Mechanism (MDM-GWO) algorithm

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Initialize the positions of grey wolves $X_w (w = 1, 2, \ldots N)$
Evaluate the function values $f(X_w)$ at X_w
Select α, β, and δ wolves
Initalize a, A and C
Initialize $t = 0$ and $FES = 0$
while ($t < Maxiter$ and $FES < maxfes$) **do**
  update parameter a using equation (15)
  for each wolf **do**
    update the position of wolves using equation (13) say Y_w
    if $f(Y_w) \leq f(X_w)$ **then**
      replace the position X_w by Y_w
    end if
    if rand() $< p_m$ **then**
      obtain a new position Z_w using equation (16)
      if $f(Z_w) \leq f(X_w)$ **then**
        replace the position X_w by Z_w
      end if
    end if
  end for
  t=t+1
end while
Return α wolf

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4. Experimental environment and results

There are many well-defined and well-analyzed benchmark sets of optimization problems to evaluate the newly designed optimization algorithms' performance. GECCO (Škvorc et al., 2019b; Molina et al., 2018; Hansen et al., 2010), CEC (Škvorc et al., 2019a; García et al., 2009; Molina et al., 2018) and 23 benchmark problems (Yao et al., 1999; Fogel, 1991; Mirjalili et al., 2014; Mirjalili and Lewis, 2016; Törn and Zilinskas, 1989; Bäck and Schwefel, 1993) etc. is a non-exhaustive list of such benchmark sets. In this paper, we have used 23 benchmark problems to evaluate the performance of the proposed MDM-GWO. In the literature, this problem set has been used widely to evaluate the performance of the population-based optimization algorithms (Bansal and Singh, 2020; Rachapudi and Devi, 2019; Zhang et al., 2020; Ibrahim et al., 2018; Dhargupta et al., 2020; Bujok, 2018b; Poláková et al., 2015; Bujok, 2018a; Wu et al., 2017). The benchmark set is representative consisting of variety of problems including unimodal, multimodal, scalable, and non-scalable problems. The description and details of these benchmark problems are presented in Table 1. 2D versions of some selected benchmark problems P1, P3, P5, P8, P9, P10, P13, P14, P15, and P22 are shown in Figure 3 (Mirjalili et al., 2014) to understand the complexities of the problems. In Table 1, the benchmark problems include 7 unimodal problems (P1-P7), 6 multimodal problems (P8-P13), and 10 multimodal problems with fixed dimensions (P14-P23). The Dimension (n) denotes the number of decision variables and Range refers the boundary of the decision variables. In Table 1, the optimal value corresponding to each well-known benchmark problem is given. The unimodal problems are to evaluate a unique feature of the algorithm called exploitation. Unlike the unimodal problems, the multimodal problems have many local optima that increase with the dimension. These problems allow testing the algorithm for its exploration ability and getting out of local optima. Another class of multimodal problems with fixed dimensions is known to be convenient in certifying exploration and exploitation ability at the same time.

In this paper, the experiments are organized in a very comprehensive manner. The experiments are carried out in three steps: First, the proposed MDM-GWO is compared with a conventional GWO and variants of GWO on 30-dimensional test problems. In the second step, a comparison between the proposed MDM-GWO, conventional GWO, and variants of GWO is performed over 50, 100, 500, and 1000-dimensional scalable test problems P1-P13. In the third step, the proposed MDM-GWO is compared with other popular metaheuristic algorithms. All these experiments are carried out under the same environment (parameter setting), using 50 search agents to conduct 5×10^4 function evaluations. During the experiments, each algorithm is independently tested 30 times on each benchmark test problem to reduce the effect of randomness. All the parameter settings and PC details are shown in Table 2 and Table 3, respectively. The experimental results are analyzed based on two criteria: 1) average (*Average*) and standard deviation (*Std.*) values of best results obtained over 30 trials of algorithm, and 2) statistical analysis through Wilcoxon rank-sum test, Friedman test and post hoc test (Holms and Hochberg). Furthermore, to analyze and compare the convergence rate and diversity, convergence curves and diversity curves are plotted in Figures $4-6$.

Table 1: Benchmark problems

Problem	Types	Dimension (n)	Range	Optimal value
P1(x)=$\sum_{i=1}^{n} x_i^2$	Unimodal	30,50,100,500,1000	[−100,100]	0
P2(x)=$\sum_{i=1}^{n} \lvert x_i^2\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$	Unimodal	30,50,100,500,1000	[−10,10]	0
P3(x)=$\sum_{i=1}^{n} \left(\sum_{j-1}^{i} x_j\right)^2$	Unimodal	30,50,100,500,1000	[−100,100]	0
P4(x)=$\max_i \{\lvert x_i\rvert, 1 \leq i \leq n\}$	Unimodal	30,50,100,500,1000	[−100,100]	0
P5(x)=$\sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$	Unimodal	30,50,100,500,1000	[−30,30]	0
P6(x)=$\sum_{i=1}^{n} ([x_i + 0.5])^2$	Unimodal	30,50,100,500,1000	[−100,100]	0
P7(x)=$\sum_{i=1}^{n} i x_i^4 + random[0,1)$	Unimodal	30,50,100,500,1000	[−1.28,1.28]	0
P8(x)=$\sum_{i=1}^{n} -x_i \sin\left(\sqrt{\lvert x_i\rvert}\right)$	Multimodal	30,50,100,500,1000	[−500,500]	-418.9829$\times D$
P9(x)=$\sum_{i=1}^{n} \left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$	Multimodal	30,50,100,500,1000	[−5.12,5.12]	0
P10(x)=$-20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	Multimodal	30,50,100,500,1000	[-32,32]	0
P11(x)=$\frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	Multimodal	30,50,100,500,1000	[−600,600]	0
P12(x)=$\frac{\pi}{n}\left\{10\sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right] + (y_n - 1)^2\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$	Multimodal	30,50,100,500,1000	[−50,50]	0
$y_i = 1 + \frac{x_i+1}{4} u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$				
P13(x)=$0.1\left\{\sin^2(3\pi x_1) + \sum_{i=1}^{n}(x_i - 1)^2\left[1 + \sin^2(3\pi x_i + 1)\right] + (x_n - 1)^2\left[1 + \sin^2(2\pi x_n)\right]\right\} +$	Multimodal	30,50,100,500,1000	[−50,50]	0
$\sum_{i=1}^{n} u(x_i, 5, 100, 4)$				
P14(x)=$\left(\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j + \sum_{i=1}^{2}(x_i - a_{ij})^6}\right)^{-1}$	Fixed-dimensional multimodal	2	$[-65, 65]$	0.998
P15(x)=$\sum_{i=1}^{11}\left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$	Fixed-dimensional multimodal	4	$[-5, 5]$	0.00030
P16(x)=$4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	Fixed-dimensional multimodal	2	$[-5, 5]$	−1.0316
P17(x)=$\left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos x_1 + 10$	Fixed-dimensional multimodal	2	$[-5, 5]$	0.398
P18(x)=$\left[1 + (x_1 + x_2 + 1)^2\left(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2\right)\right]$	Fixed-dimensional multimodal	2	$[-2, 2]$	3
$\left[30 + (2x_1 - 3x_2)^2\left(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2\right)\right]$				
P19(x)=$-\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2\right)$	Fixed-dimensional multimodal	3	$[1, 3]$	−3.86
P20(x)=$-\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2\right)$	Fixed-dimensional multimodal	6	$[0, 1]$	−3.32
P21(x)=$-\sum_{i=1}^{5}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	Fixed-dimensional multimodal	4	$[0, 10]$	−10.1532
P22(x)=$-\sum_{i=1}^{7}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	Fixed-dimensional multimodal	4	$[0, 10]$	−10.4028
P23(x)=$-\sum_{i=1}^{10}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	Fixed-dimensional multimodal	4	$[0.10]$	−10.5363

Table 2: Parameter setting of the various algorithms

S.No	Algorithms	Parameter setting
1	GWO	$a = [2, 0]$
2	OGWO	
3	RW-GWO	
4	MGWO	User defined parameters selected from the original papers
5	GWO-XOBL	
6	WF-GWO	
7	BBO	I = 1, E = 1, m_{max} = 0.005, keep = 2
8	PSO	c_1 and c_2=2, v_{max}=ub, v_{min}=lb inertia weight is linearly decreasing from 0.9 to 0.4
9	GSA	gravitational constant G = 100, decreasing coefficient β = 20
10	SSA	$c_1 = 2 \times e^{-(\frac{4l}{L})^2}$
11	CS	$P_a = 0.25$ and $\kappa = 1.5$
12	HHO	$E0 \in [-1, 1]$
13	CMA-ES	$alpha_{mu} = 2$

Table 3: The detailed settings of the utilized system for experimentation

Name	**Setting**
CPU	Intel Core(TM) i5 processor
RAM	8 GB
Software	MATLAB R2014a
Operating system	Windows 10

4.1. Analysis of different strategies

In this subsection, to verify the impact of each strategy in the MDM-GWO, four different combinations are added in the GWO algorithm such as Strategy-1, Strategy-2, Strategy-3, and Strategy-4. In Strategy-1, only a modified control parameter is added to the GWO algorithm. In Strategy-2, the levy-flight mutation is added to the GWO algorithm, and In Strategy-3, the combination of the modified control parameter and levy-flight mutation is added to the GWO algorithm. Strategy-4 or MDM-GWO means that all strategies are added to the GWO algorithm. Same benchmark problems (P1-P23) have been used to verify the impact of each strategy. The parameter settings are as same as in section 4, and each benchmark problem runs 30 times, individually. The specific experimental results are shown in Table 4. Besides, the *Average* and standard deviation (*Std.*) values of the objective functions and the statistical analysis through the Wilcoxon rank-sum test are reported in Table 4. In Table 4, '+' means that the proposed MDM-GWO is better than Strategy-1, Strategy-2, and Strategy-3, '-' indicates that MDM-GWO is worse than Strategy-1, Strategy-2, and Strategy-3,

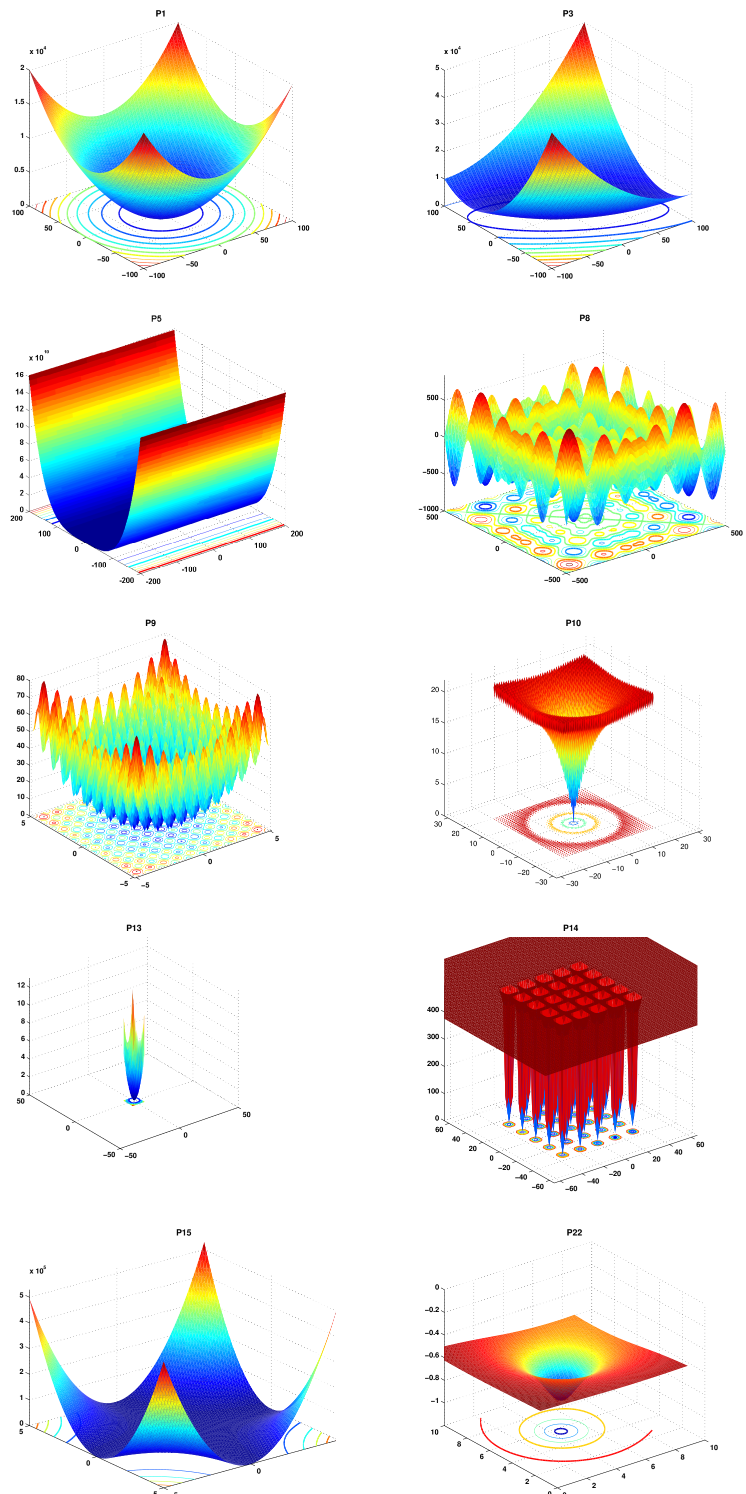

Figure 3: 2D versions of some selected benchmark problems

and '≈' represents that MDM-GWO is the same as Strategy-1, Strategy-2, and Strategy-3. In the first four unimodal benchmark problems, P1-P4, the experimental results of Table 4 show that the proposed MDM-GWO has performed better than other Strategy-1, Strategy-2, and Strategy-3. Besides, Strategy-2 is the second-best optimizer in these benchmark problems. In problem P6, Strategy-3 has performed better than other strategies and MDM-GWO. The proposed MDM-GWO has performed better in problems P7-P11. In the problem P9 and P11, the MDM-GWO achieves the optima value (0) and outperforms Strategy-1, Strategy-2, and Strategy-3. In problems P12-P18 and P22, Strategy-3 is better, and in problem P19, Strategy-1, Strategy-2, Strategy-3, and MDM-GWO perform the same in terms of average and standard deviation. Strategy-2 performs better for problem P20, and for problem P23, Strategy-1 is a better optimizer. The proposed MDM-GWO achieves optimal value for problem P21. From the results reported in Table 4, it can be concluded that MDM-GWO outperforms Strategy-1, Strategy-2, and Strategy-3. Hence, a modified search equation effectively improves the results compared to using levy flight or modified control parameter only.

4.2. Comparison with Conventional GWO and variants of GWO

In this subsection, the performance of the MDM-GWO is compared with conventional GWO (Mirjalili et al., 2014) and the variants of the GWO such as MGWO (Mittal et al., 2016), OGWO (Pradhan et al., 2018), RW-GWO (Gupta and Deep, 2019), GWO-XOBL (Singh and Bansal, 2020) and WF-GWO (Rodríguez et al., 2017). The parameter setting for the comparison is taken the same, i.e., 50 population size and 5×10^4 function evaluations. With this parameter setting, Table 5 reports the *Average* and *Std.* of the best value obtained over 30 trials of all the algorithms. The best results are highlighted in bold. Since the unimodal problems (P1-P7) possess unique (global) optimal solutions, these functions verify the exploitation efficiency of the proposed algorithm. From the results of Table 5, it is evident that the *Average* and *Std.* of the proposed MDM-GWO are better than the conventional GWO and the variants of the GWO for unimodal problems P1 and P3-P7. In problem P1, GWO-XOBL and the proposed MDM-GWO have achieved global optimal solution (0), and GWO-XOBL performs better for problem P2. For multimodal problems P9-P13, the proposed MDM-GWO performs significantly better than the conventional GWO and the variants of the GWO algorithm. In P8, GWO-XOBL is better in terms of *Average*, and

Table 4: Obtained results by Strategy-1, Strategy-2, Strategy-3, and Strategy-4 on 23 well-known benchmark problems

Problem		Strategy-1	Strategy-2	Strategy-3	Strategy-4(MDM-GWO)
P1	**Average**	8.34E-37	1.74E-76	9.12E-37	7.07E-307
	Std.	1.68E-36	4.68E-76	1.12E-36	0.00E+00
	outcome	+	+	+	
P2	**Average**	8.13E-22	1.26E-44	9.84E-22	5.07E-152
	Std.	5.15E-22	1.09E-44	5.15E-22	2.78E-151
	outcome	+	+	+	
P3	**Average**	1.95E-07	3.64E-16	1.26E-07	1.74E-273
	Std.	4.56E-07	1.90E-15	3.33E-07	0.00E+00
	outcome	+	+	+	
P4	**Average**	4.41E-08	2.06E-16	2.79E-08	7.14E-149
	Std.	9.58E-08	3.59E-16	3.01E-08	3.11E-148
	outcome	+	+	+	
P5	**Average**	2.64E+01	2.25E+01	1.04E+01	1.62E+00
	Std.	6.78E-01	7.32E+00	1.26E+01	1.22E+00
	outcome	+	+	+	
P6	**Average**	2.94E-01	5.23E-03	7.73E-09	8.98E-03
	Std.	2.59E-01	5.98E-04	7.73E-09	2.39E-03
	outcome	+	+	-	
P7	**Average**	1.29E-03	6.32E-04	1.31E-03	1.68E-04
	Std.	5.83E-04	2.70E-04	7.34E-04	1.72E-04
	outcome	+	+	+	
P8	**Average**	-5.97E+03	-5.99E+03	-5.92E+03	-7.48E+03
	Std.	1.23E+03	6.20E+02	1.13E+03	1.01E+03
	outcome	+	+	+	
P9	**Average**	4.24E+00	5.20E+00	3.37E+00	0.00E+00
	Std.	5.64E+00	7.31E+00	4.77E+00	0.00E+00
	outcome	+	+	+	
P10	**Average**	1.97E-14	9.30E-15	8.11E-02	1.95E-15
	Std.	3.86E-15	5.94E-15	4.44E-01	1.66E-15
	outcome	+	+	+	
P11	**Average**	1.17E-03	3.27E-03	9.47E-04	0.00E+00
	Std.	3.79E-03	6.41E-03	3.60E-03	0.00E+00
	outcome	+	+	+	
P12	**Average**	2.15E-02	1.01E-03	1.07E-04	1.56E-03
	Std.	1.07E-02	2.09E-03	4.41E-04	4.97E-04
	outcome	+	-	-	
P13	**Average**	2.88E-01	2.70E-02	6.69E-03	2.50E-02
	Std.	1.40E-01	5.06E-02	2.17E-02	1.49E-02
	outcome	+	≈	-	
P14	**Average**	2.11E+00	2.83E+00	2.90E+00	9.98E-01
	Std.	2.74E+00	2.85E+00	3.26E+00	2.36E-10
	outcome	+	+	+	
P15	**Average**	3.77E-03	2.39E-03	3.06E-03	7.83E-04
	Std.	7.55E-03	6.10E-03	6.91E-03	3.07E-04
	outcome	+	+	+	
P16	**Average**	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Std.	1.21E-14	2.22E-07	9.63E-13	6.13E-08
	outcome	-	+	-	
P17	**Average**	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Std.	8.34E-13	1.21E-05	2.42E-11	2.60E-07
	outcome	-	+	-	-
P18	**Average**	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Std.	1.25E-05	3.23E-06	1.85E-08	2.33E-06
	outcome	+	≈	-	
P19	**Average**	-3.00E-01	-3.00E-01	-3.00E-01	-3.00E-01
	Std.	2.26E-16	2.26E-16	2.26E-16	2.26E-16
	outcome	≈	≈	≈	
P20	**Average**	-3.26E+00	-3.27E+00	-3.25E+00	-3.23E+00
	Std.	6.76E-02	6.06E-02	6.74E-02	6.73E-02
	outcome	-	-	-	
P21	**Average**	-9.45E+00	-9.22E+00	-9.65E+00	-1.01E+01
	Std.	1.83E+00	1.91E+00	1.54E+00	6.45E-03
	outcome	+	+	+	
P22	**Average**	-1.02E+01	-1.02E+01	-1.04E+01	-1.04E+01
	Std.	9.70E-01	6.62E-01	9.91E-08	5.92E-03
	outcome	+	+	-	
P23	**Average**	-1.05E+01	-1.01E+01	-9.73E+00	-1.05E+01
	Std.	2.20E-09	1.31E+00	2.14E+00	8.17E-03
	outcome	-	+	+	

OGWO is better in *Std*. For the multimodal with fixed-dimensional problems (P14-P23), the proposed MDM-GWO performs better for P14, P16-P19, and P21-P23. OGWO performs better in P15, and GWO-XOBL performs better in P20.

The convergence rate in the proposed MDM-GWO is compared through convergence curves for selected well-known benchmark problems P1, P3, P5, P8, P9, P10, P13, P14, P15, and P22 plotted in Figure 4. The curves are drawn for the best value of the objective function in each iteration in these figures. The horizontal axis represents the maximum number of function evaluations, and the vertical axis indicates the best value of the objective function.

4.2.1. Diversity analysis

In this section, the proposed method, MDM-GWO, is evaluated for its capability of diversified search. This is done by the radius of swarm (Olorunda and Engelbrecht, 2008). The radius of the swarm is defined as the maximum distance between the swarm center and any grey wolf position in the swarm. The radius can be calculated by the following formula:

$$Rd = \max_{i \in [1,N]} \left(\sqrt{\sum_{k=1}^{Dimension(n)} (X_{i,k} - \bar{X_k})^2} \right)$$

where, $X_{i,k}$ is the k^{th} dimension of the i^{th} grey wolf position and $\bar{X_k}$ is the k^{th} dimension of the swarm center position.

The obtained diversity curves for selected well-known classical problems P1, P3, P5, P8, P9, P10, P13, P14, P15, and P22 are shown in Figure 5. From these figures, it can be seen that at the initial stage, the maximum radius is higher in the MDM-GWO than the conventional GWO, which shows that the proposed MDM-GWO has a good exploration capability of search agent. The reason for this enhanced diversity is the additional exploratory search strategy based on new search equations and mutation strategies applied in the MDM-GWO.

4.3. Computational Complexity

In these metaheuristic algorithms, the computational complexity determines how long the method needs to figure out the final result. The computational complexities of the conventional GWO and MDM-GWO

Table 5: Comparison of results obtained by MDM-GWO with selected variants of GWO on 23 well-known benchmark problems

Problem		GWO	OGWO	RW-GWO	MGWO	GWO-XOBL	WF-GWO	MDM-GWO
P1	**Average**	3.72E-77	1.90E-91	1.27E-74	8.58E-35	**0.00E+00**	6.03E-76	**0.00E+00**
	Std.	8.07E-77	8.14E-91	1.66E-74	2.01E-34	**0.00E+00**	1.11E-75	**0.00E+00**
P2	**Average**	6.59E-45	1.38E-54	3.78E-43	3.27E-20	**1.01E-177**	9.08E-44	9.77E-157
	Std.	6.14E-45	1.88E-54	4.41E-43	1.32E-20	**0.00E+00**	1.03E-43	3.21E-156
P3	**Average**	8.09E-17	4.00E-15	4.31E-11	4.71E-10	5.45E-216	5.09E-18	**3.21E-277**
	Std.	3.56E-16	2.10E-14	2.04E-10	1.13E-09	**0.00E+00**	9.63E-18	**0.00E+00**
P4	**Average**	1.17E-16	1.21E-29	6.69E-14	1.71E-06	4.86E-134	1.61E-16	**9.63E-149**
	Std.	2.13E-16	1.21E-29	8.26E-14	1.37E-06	8.33E-134	3.72E-16	**3.32E-148**
P5	**Average**	2.66E+01	2.68E+01	2.57E+01	2.81E+01	2.55E+01	2.62E+01	**1.24E+00**
	Std.	7.36E-01	6.51E-01	**5.37E-01**	9.22E-01	5.46E-01	6.88E-01	1.24E+00
P6	**Average**	3.62E-01	6.34E-01	5.31E-02	2.80E+00	1.83E-02	4.53E-01	**9.34E-03**
	Std.	2.84E-01	3.13E-01	8.56E-02	7.04E-01	6.17E-02	3.30E-01	**2.59E-03**
P7	**Average**	7.28E-04	2.34E-04	1.28E-03	7.43E-03	1.72E-03	7.66E-04	**2.34E-04**
	Std.	4.08E-04	1.18E-04	4.96E-04	3.20E-03	1.03E-03	2.88E-04	**2.19E-04**
P8	**Average**	-6.58E+03	-3.81E+03	-8.69E+03	-5.59E+03	**-9.00E+03**	-6.31E+03	-7.89E+03
	Std.	7.92E+02	**3.46E+02**	3.98E+02	1.00E+03	5.11E+02	9.69E+02	1.43E+03
P9	**Average**	4.05E+00	**0.00E+00**	1.25E+01	1.13E+02	**0.00E+00**	1.31E+00	**0.00E+00**
	Std.	6.43E+00	0.00E+00	8.80E+00	3.17E+01	0.00E+00	2.89E+00	**0.00E+00**
P10	**Average**	7.88E-15	7.28E-15	7.99E-15	2.27E+00	4.44E-15	7.99E-15	**1.72E-15**
	Std.	6.49E-16	1.45E-15	0.00E+00	1.76E+00	0.00E+00	0.00E+00	1.53E-15
P11	**Average**	1.49E-03	3.62E-04	1.89E-03	4.58E-03	0.00E+00	2.20E-03	**0.00E+00**
	Std.	4.02E-03	1.98E-03	5.60E-03	6.65E-03	0.00E+00	5.30E-03	**0.00E+00**
P12	**Average**	2.39E-02	4.18E-02	4.62E-03	1.96E+01	3.91E-03	2.34E-02	**1.56E-03**
	Std.	1.44E-02	1.93E-02	6.81E-03	1.62E+01	5.43E-03	9.50E-03	**4.79E-04**
P13	**Average**	3.18E-01	5.60E-01	5.16E-02	2.05E+00	9.74E-02	3.49E-01	**3.33E-02**
	Std.	1.94E-01	1.93E-01	5.97E-02	7.87E-01	1.22E-01	1.88E-01	**2.04E-02**
P14	**Average**	4.32E+00	2.83E+00	9.98E-01	2.41E+00	9.98E-01	4.26E+00	**9.98E-01**
	Std.	4.35E+00	3.47E+00	**1.04E-11**	2.45E+00	7.00E-11	4.23E+00	1.99E-10
P15	**Average**	3.74E-03	**3.95E-04**	4.92E-04	6.02E-04	1.14E-03	4.40E-03	8.96E-04
	Std.	7.57E-03	2.84E-04	3.76E-04	**2.79E-04**	3.65E-03	8.10E-03	3.29E-04
P16	**Average**	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Std.	8.86E-10	4.88E-06	1.43E-09	4.28E-04	7.04E-07	1.46E-05	2.55E-08
P17	**Average**	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Std.	5.75E-08	1.01E-04	7.50E-08	5.03E-07	3.27E-07	3.89E-08	3.43E-07
P18	**Average**	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Std.	2.39E-06	3.21E-06	4.95E-07	3.15E-04	7.31E-08	4.17E-06	2.54E-06
P19	**Average**	-3.00E-01	-3.00E-01	-3.00E-01	-3.00E-01	-3.00E-01	-3.00E-01	-3.00E-01
	Std.	2.26E-16	2.26E-16	2.26E-16	2.26E-16	2.26E-16	2.26E-16	2.26E-16
P20	**Average**	-3.25E+00	-3.22E+00	-3.26E+00	-3.25E+00	**-3.26E+00**	-3.25E+00	-3.22E+00
	Std.	6.80E-02	7.44E-02	6.12E-02	6.03E-02	6.06E-02	8.23E-02	6.10E-02
P21	**Average**	-9.48E+00	-8.28E+00	-9.29E+00	-6.80E+00	-1.01E+01	-9.27E+00	**-1.01E+01**
	Std.	1.75E+00	1.90E+00	1.92E+00	3.48E+00	5.00E-03	2.03E+00	**3.32E-03**
P22	**Average**	-1.02E+01	-8.49E+00	-1.02E+01	-6.88E+00	-1.02E+01	**-1.04E+01**	-1.04E+01
	Std.	9.70E-01	1.52E+00	1.34E+00	3.65E+00	9.69E-01	**9.44E-05**	7.15E-03
P23	**Average**	-1.05E+01	-8.68E+00	**-1.05E+01**	-7.86E+00	-1.05E+01	-1.04E+01	-1.05E+01
	Std.	5.84E-05	1.68E+00	**4.39E-05**	3.62E+00	5.42E-03	9.79E-01	6.32E-03

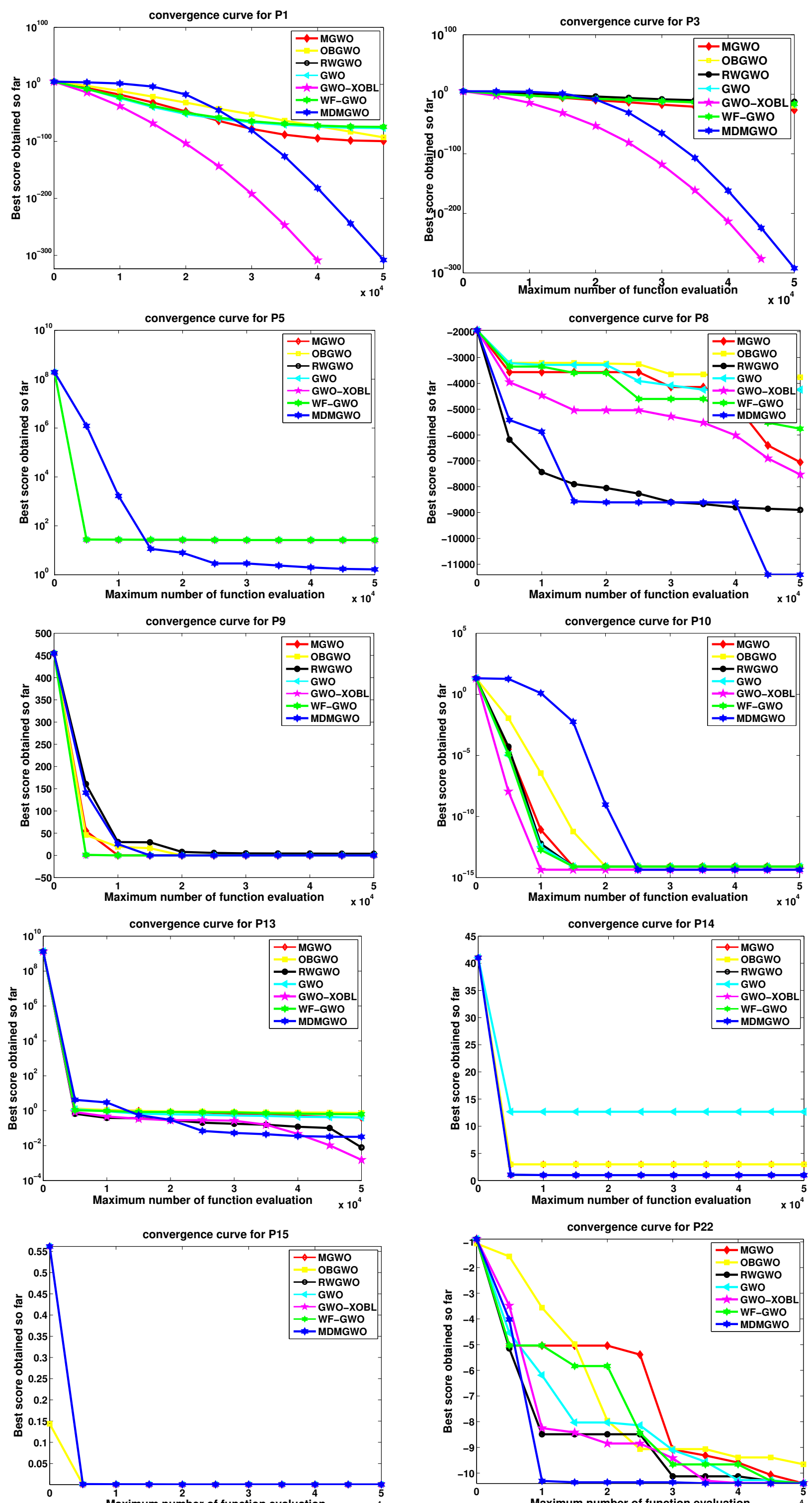

Figure 4: Convergence curves

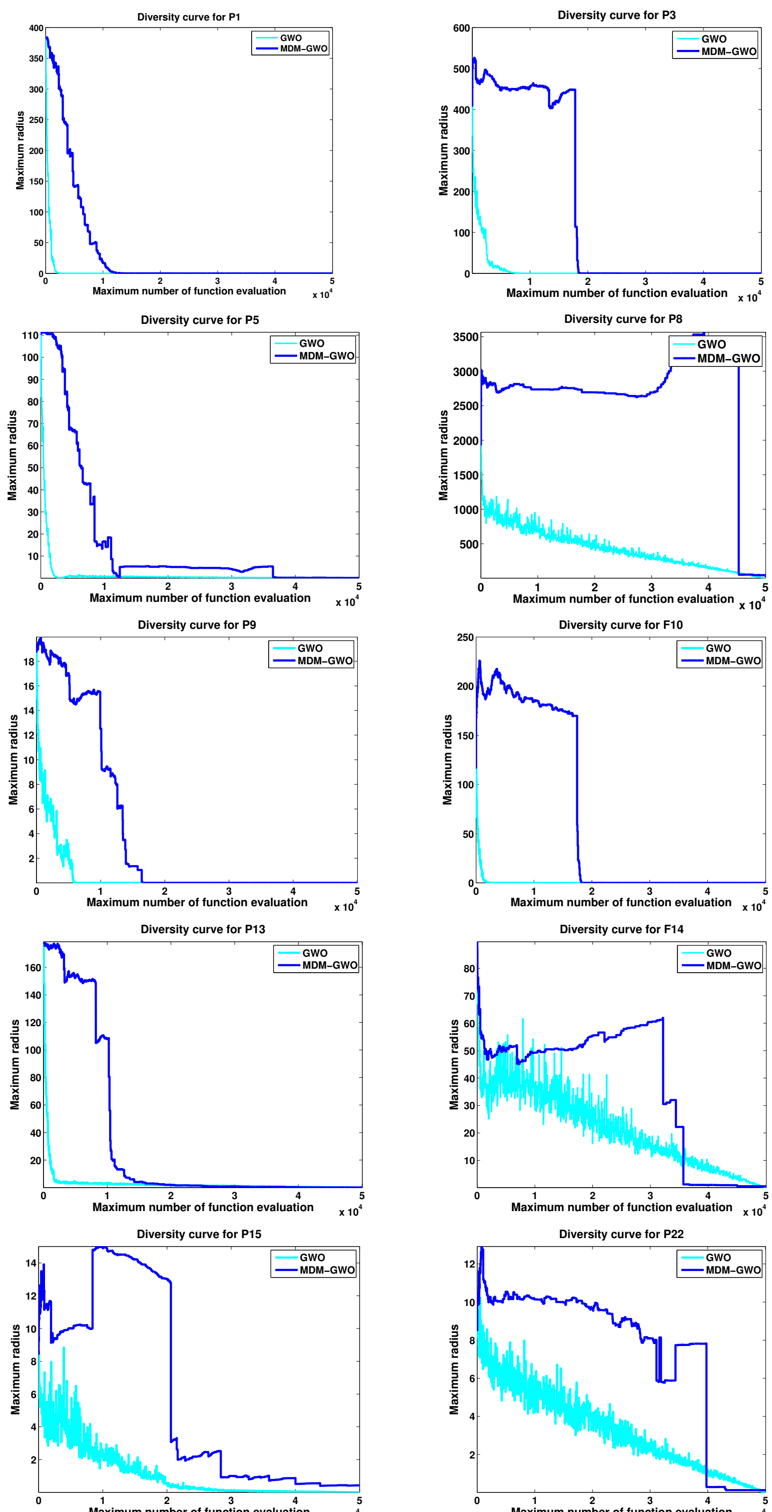

Figure 5: Diversity curves

algorithms are calculated in terms of a big-O notation. The algorithms' computational complexity depends on initialization, fitness evaluation, position update mechanism, mutation, and greedy selection approach. In both the algorithms, the computational complexity of initialization and fitness evaluation is $O(N \times D)$ and $O(N)$ time. In GWO, the position update mechanism is $O(N \times D)$ time. Therefore, total computational complexity is $O(N \times D \times Maxiter)$, where N is the number of grey wolves, D is the search space dimension, and $Maxiter$ is the maximum number of iterations allowed. In MDM-GWO, the modified search mechanism's computational complexity is $O(N \times D)$ time. Also, the computational complexity of the mutation is $O(N \times D)$ time, and the greedy selection approach is $O(N)$ time. Therefore, the total computational complexity for MDM-GWO is $O(N \times D \times Maxiter)$. Note that the computational complexity of the MDM-GWO and the conventional GWO is the same.

4.4. Experiments on large-scale problems

In order to analyze the performance of the proposed MDM-GWO on the optimization problems with higher difficulty levels, numerical experiments are performed on 50, 100, 500, and 1000-dimension problems of scalable problems P1-P13. The results of the proposed MDM-GWO are compared with conventional GWO, MGWO, OGWO, RW-GWO, GWO-XOBL, and WF-GWO algorithms. All the algorithms were run with the same conditions. The number of search agents and the maximum function evaluation are fixed at 50 and 5×10^4, respectively. The average of the best value ($Average$) and standard deviation (*Std.*) obtained by the proposed MDM-GWO and other algorithms are recorded. The specific results are reported in Tables 6 and 7, respectively. As can be seen from Tables 6 and 7, the proposed MDM-GWO has a better performance compared with other algorithms for 50, 100, 500, and 1000 dimension problems. For the 50 dimension problem, MDM-GWO is a better optimizer compared to other competitors in problems P1, P3-P7, and P9-P13 but in P2 and P8, GWO-XOBL and RW-GWO perform better. For the 100, 500, and 1000 dimensions, the MDM-GWO beats other competitors in all the problems P1-P13 but in P5, MGWO is better in terms of standard deviation. The convergence curves for some selected well-known benchmark problems P1, P3, P5, P7, P8, P9, P10, and P13 are depicted in Figure 6. In these Figures, GWO1 and MDM-GWO1 are for the 50 dimension problem, GWO2 and MDM-GWO2 are for the 100 dimension problem, GWO3 and MDM-GWO3 are for 500 dimension problem, and GWO4 and MDM-GWO4 are for 500 dimension problem, respectively. From these Figures, it can be concluded that the MDM-GWO has faster convergence while maintaining a better accuracy level compared to conventional GWO for higher dimensions. The above analysis suggests that the applied strategies are suitable for complex problems, and the performance of MDM-GWO becomes more robust for higher dimension problems.

4.5. Statistical analysis

To determine whether the algorithm MDM-GWO is significantly better than the competitors, a non-parametric test, Wilcoxon rank-sum test (Derrac et al., 2011) is used for 30, 50, 100, 500, and 1000 dimensions. The statistical outcomes with a 5% significance level are shown in Tables 8-10. In these tables, $+/-/\approx$ represents that MDM-GWO is better/worse/uniform as the algorithms used for comparison, respectively.

Furthermore, the Friedman test (Carrasco et al., 2020; Derrac et al., 2014) which is a non-parametric and multiple-comparison test is used to check whether there is a significant differences between two or more algorithms and used to assess the significant difference in five different dimensions (Dim =30, 50, 100, 500, and 1000). Table 11 shows the statistical results of five different dimensions ordered in terms of average ranking produced by the Friedman test. Note that the proposed MDM-GWO always received the first rank under different dimensions. The calculated Friedman value is 52.9 with 6 degrees of freedom (distributed according to χ^2), and the critical value is 12.59 with a 5% level of significance. It is clear that 52.9>12.59 and the obtained p-value of Friedman test is 1.20E-09 which is less than the 0.05. Hence, there is a significant difference between the MDM-GWO and the other algorithms. Additionally, in order to detect the difference between the comparison methods, we have also performed post-hoc tests, the Holms and Hochberg statistical tests. Table 12 presents the adjusted p-value and 0.05/rank for each comparison pair between MDM-GWO and MGWO, OGWO, GWO, RW-GWO, GWO-XOBL, and WF-GWO, rank defined as the lower is the better and will be ranked first. In Holms and Hochberg statistical tests, if adjusted p-value $< 0.05/\text{rank}$, this means that there is a significant difference between MDM-GWO and other algorithms. For 30 dimension test problems, the proposed MDM-GWO is significantly better than GWO, OGWO, WF-GWO, and MGWO, for 50 dimension test problems, MDM-GWO is significantly better than RW-GWO, GWO, OGWO, WF-GWO, and

Table 6: Comparison of results obtained by proposed MDM-GWO and variants of GWO algorithm on scalable benchmark problems with 50 and 100 dimension

Problem		GWO	OGWO	RW-GWO	MGWO	GWO-XOBL	Weight-GWO	MDMGWO
For 50 dimension								
P1	**Average**	2.23E-56	2.88E-72	4.42E-54	1.31E-71	1.92E-297	4.94E-56	**4.63E-308**
	Std.	3.56E-56	9.25E-72	5.51E-54	3.02E-71	0.00E+00	6.16E-56	**0.00E+00**
P2	**Average**	1.82E-33	2.85E-43	5.36E-32	1.82E-42	**1.45E-160**	1.23E-31	2.33E-155
	Std.	1.84E-33	3.84E-43	4.50E-32	1.60E-42	**2.58E-160**	2.05E-31	1.18E-154
P3	**Average**	2.21E-06	1.65E-04	2.19E-02	2.72E-07	2.45E-186	3.99E-06	**1.75E-273**
	Std.	4.05E-06	8.06E-04	5.37E-02	8.57E-07	0.00E+00	8.97E-06	**0.00E+00**
P4	**Average**	1.21E-09	4.71E-22	5.73E-06	1.29E-13	4.73E-119	8.71E-10	**6.57E-147**
	Std.	2.07E-09	1.14E-21	8.72E-06	4.53E-13	1.52E-118	1.40E-09	**3.49E-146**
P5	**Average**	4.67E+01	4.70E+01	4.63E+01	4.66E+01	4.55E+01	4.66E+01	**3.47E+00**
	Std.	1.02E+00	8.75E-01	**5.99E-01**	6.88E-01	8.15E-01	7.88E-01	2.2654E+00
P6	**Average**	1.59E+00	2.43E+00	6.22E-01	1.47E+00	4.91E-01	1.56E+00	**3.60E-02**
	Std.	5.47E-01	5.21E-01	3.65E-01	5.20E-01	3.51E-01	6.03E-01	**7.17E-03**
P7	**Average**	1.05E-03	3.62E-04	2.21E-03	9.11E-04	2.81E-03	1.14E-03	**1.86E-04**
	Std.	3.58E-04	2.38E-04	1.03E-03	5.02E-04	1.33E-03	3.82E-04	**1.84E-04**
P8	**Average**	-8.84E+03	-4.76E+03	**-1.36E+04**	-8.53E+03	-1.29E+04	-9.46E+03	-1.20E+04
	Std.	2.34E+03	4.10E+02	6.79E+02	2.52E+03	8.80E+02	2.14E+03	1.33E+03
P9	**Average**	5.70E+00	0.00E+00	1.66E+01	7.05E-01	0.00E+00	5.03E+00	**0.00E+00**
	Std.	1.09E+01	0.00E+00	8.51E+00	2.81E+00	0.00E+00	7.06E+00	**0.00E+00**
P10	**Average**	1.34E-14	8.23E-15	1.49E-14	8.23E-15	4.44E-15	1.36E-14	**1.24E-15**
	Std.	3.06E-15	1.30E-15	2.27E-15	1.30E-15	**0.00E+00**	2.90E-15	1.08E-15
P11	**Average**	9.85E-04	0.00E+00	1.70E-03	2.98E-04	0.00E+00	7.51E-04	**0.00E+00**
	Std.	3.02E-03	0.00E+00	3.89E-03	1.63E-03	0.00E+00	2.86E-03	**0.00E+00**
P12	**Average**	5.85E-02	9.67E-02	1.92E-02	6.50E-02	1.23E-02	5.60E-02	**1.79E-03**
	Std.	1.99E-02	3.38E-02	8.70E-03	6.19E-02	1.07E-02	2.02E-02	**4.99E-04**
P13	**Average**	1.33E+00	1.96E+00	5.04E-01	1.19E+00	5.42E-01	1.45E+00	**6.39E-02**
	Std.	3.71E-01	3.44E-01	2.97E-01	2.70E-01	2.55E-01	2.88E-01	**2.62E-02**
For 100 dimension								
P1	**Average**	3.66E-37	4.70E-55	1.00E-34	1.26E-46	4.29E-267	4.33E-37	**1.53E-302**
	Std.	6.19E-37	7.99E-55	1.26E-34	1.34E-46	0.00E+00	4.07E-37	**0.00E+00**
P2	**Average**	1.01E-22	3.38E-33	1.56E-21	2.29E-28	4.95E-146	1.21E-18	**1.15E-152**
	Std.	5.54E-23	2.85E-33	6.10E-22	1.44E-28	1.10E-145	4.48E-18	**5.48E-152**
P3	**Average**	5.57E+01	1.34E+03	1.12E+03	6.15E+01	1.35E-160	2.60E+01	**1.02E-264**
	Std.	6.25E+01	2.32E+03	9.64E+02	9.04E+01	7.42E-160	5.83E+01	**0.00E+00**
P4	**Average**	4.93E-03	7.92E-15	2.42E+00	3.43E-03	1.58E-106	1.01E-02	**4.06E-145**
	Std.	4.81E-03	2.56E-14	1.84E+00	1.04E-02	4.85E-106	2.68E-02	**1.26E-144**
P5	**Average**	9.72E+01	9.73E+01	9.66E+01	9.67E+01	9.63E+01	9.73E+01	**7.65E+00**
	Std.	8.43E-01	8.19E-01	8.09E-01	**7.45E-01**	1.27E+00	8.68E-01	1.10E+01
P6	**Average**	7.01E+00	9.64E+00	4.56E+00	7.21E+00	3.51E+00	7.12E+00	**1.08E-01**
	Std.	9.30E-01	8.81E-01	7.65E-01	9.62E-01	8.38E-01	1.13E+00	**2.47E-02**
P7	**Average**	1.96E-03	5.45E-04	4.46E-03	1.47E-03	4.74E-03	2.49E-03	**2.34E-04**
	Std.	7.76E-04	3.90E-04	1.16E-03	4.57E-04	1.90E-03	1.08E-03	**1.98E-04**
P8	**Average**	-1.63E+04	-6.52E+03	-2.36E+04	-1.59E+04	-2.09E+04	-1.70E+04	**-2.45E+04**
	Std.	4.02E+03	**6.08E+02**	1.00E+03	4.46E+03	1.18E+03	4.25E+03	2.86E+03
P9	**Average**	5.42E+00	0.00E+00	2.88E+01	3.79E-15	0.00E+00	6.15E+00	**0.00E+00**
	Std.	7.27E+00	0.00E+00	1.80E+01	2.08E-14	0.00E+00	1.17E+01	**0.00E+00**
P10	**Average**	3.56E-14	1.33E-14	4.26E-14	1.63E-14	4.56E-15	3.71E-14	**1.95E-15**
	Std.	3.69E-15	3.06E-15	2.27E-15	2.69E-15	6.49E-16	4.21E-15	**1.66E-15**
P11	**Average**	1.63E-03	5.32E-04	0.00E+00	5.02E-04	0.00E+00	2.32E-03	**0.00E+00**
	Std.	5.03E-03	2.92E-03	0.00E+00	2.75E-03	0.00E+00	5.60E-03	**0.00E+00**
P12	**Average**	1.72E-01	2.66E-01	8.66E-02	1.66E-01	7.34E-02	2.29E-01	**1.88E-03**
	Std.	4.89E-02	4.43E-02	3.10E-02	4.16E-02	2.34E-02	8.83E-02	**3.28E-04**
P13	**Average**	5.61E+00	6.63E+00	4.49E+00	5.42E+00	3.82E+00	5.43E+00	**1.39E-01**
	Std.	4.24E-01	2.79E-01	6.50E-01	3.67E-01	5.73E-01	3.37E-01	**2.97E-02**

Table 7: Comparison of results obtained by proposed MDM-GWO and variants of GWO algorithm on scalable benchmark problems with 500 and 1000 dimension

Problem		**GWO**	**OGWO**	**RW-GWO**	**MGWO**	**GWO-XOBL**	**Weight-GWO**	**MDM-GWO**
For 500 dimension								
P1	**Average**	2.25E-15	1.89E-35	6.07E-13	8.56E-19	8.23E-237	2.79E-15	**1.19E-295**
	Std.	1.24E-15	1.64E-35	3.29E-13	4.24E-19	0.00E+00	1.49E-15	**0.00E+00**
P2	**Average**	1.05E-09	3.39E-21	9.14E-09	4.98E-12	4.20E-130	5.43E+02	**4.20E-151**
	Std.	2.71E-10	1.62E-21	2.19E-09	1.53E-12	6.12E-130	6.80E+02	**1.48E-150**
P3	**Average**	2.55E+05	8.68E+05	5.40E+05	3.44E+05	1.87E-126	2.72E+05	**3.98E-264**
	Std.	5.69E+04	1.89E+05	8.39E+04	8.50E+04	1.02E-125	7.10E+04	**0.00E+00**
P4	**Average**	5.48E+01	3.12E+01	5.77E+01	5.85E+01	1.59E-77	5.46E+01	**2.06E-141**
	Std.	5.24E+00	3.37E+01	3.11E+00	4.21E+00	4.12E-77	5.40E+00	**8.12E-141**
P5	**Average**	4.97E+02	4.98E+02	4.97E+02	4.97E+02	4.97E+02	4.97E+02	**5.00E+01**
	Std.	2.91E-01	**1.57E-01**	1.81E-01	2.76E-01	4.00E-01	2.64E-01	5.89E+01
P6	**Average**	8.61E+01	9.76E+01	7.97E+01	8.91E+01	7.51E+01	8.57E+01	**7.18E-01**
	Std.	2.17E+00	1.10E+00	1.96E+00	1.73E+00	2.22E+00	1.67E+00	**1.16E-01**
P7	**Average**	1.07E-02	9.64E-04	2.26E-02	7.60E-03	8.20E-03	1.03E-02	**1.97E-04**
	Std.	3.70E-03	3.62E-04	4.75E-03	2.08E-03	3.26E-03	3.04E-03	**1.62E-04**
P8	**Average**	-6.50E+04	-1.48E+04	-6.37E+04	-4.75E+04	-6.96E+04	-6.61E+04	**-1.27E+05**
	Std.	1.45E+04	**1.24E+03**	2.85E+03	2.26E+04	3.55E+03	1.46E+04	1.02E+04
P9	**Average**	1.59E+01	6.06E-14	7.50E+01	3.54E-01	0.00E+00	1.45E+01	**0.00E+00**
	Std.	1.07E+01	2.31E-13	3.87E+01	1.94E+00	0.00E+00	1.32E+01	**0.00E+00**
P10	**Average**	2.29E-09	2.37E-14	3.31E-08	4.37E-11	5.03E-15	2.29E-09	**1.72E-15**
	Std.	5.99E-10	4.73E-15	9.35E-09	2.17E-11	**1.35E-15**	6.20E-10	1.53E-15
P11	**Average**	6.14E-04	1.04E-16	3.67E-03	1.56E-03	0.00E+00	3.83E-03	**0.00E+00**
	Std.	3.36E-03	2.82E-17	1.23E-02	5.92E-03	0.00E+00	1.23E-02	**0.00E+00**
P12	**Average**	6.64E-01	8.02E-01	6.01E-01	6.74E-01	4.99E-01	6.51E-01	**1.57E-03**
	Std.	3.80E-02	2.28E-02	4.83E-02	3.59E-02	3.77E-02	3.21E-02	**2.47E-04**
P13	**Average**	4.51E+01	4.74E+01	4.72E+01	4.48E+01	4.21E+01	4.51E+01	**5.40E-01**
	Std.	5.93E-01	4.17E-01	1.64E+00	5.72E-01	8.21E-01	8.50E-01	**9.40E-02**
For 1000 dimension								
P1	**Average**	9.68E-11	4.40E-31	2.27E-08	2.87E-13	2.47E-230	1.25E-10	**3.79E-294**
	Std.	3.98E-11	5.52E-31	1.06E-08	1.25E-13	0.00E+00	5.51E-11	**0.00E+00**
P2	**Average**	7.09E-07	5.98E-12	1.38E-04	1.184E-08	5.59E-155	3.75E-04	**6.53E-148**
	Std.	1.17E-07	2.89E-10	9.48E-04	2.26E-09	2.49E-156	1.86E-04	**2.25E-147**
P3	**Average**	1.36E+06	3.65E+06	2.36E+06	1.51E+06	6.26E-122	1.35E+06	**3.70E-263**
	Std.	1.71E+05	8.26E+05	3.06E+05	2.90E+05	3.42E-121	2.30E+05	**0.00E+00**
P4	**Average**	7.08E+01	8.88E+01	7.08E+01	7.39E+01	4.71E-65	7.12E+01	**1.62E-139**
	Std.	3.14E+00	6.35E+00	3.65E+00	2.32E+00	2.23E-64	2.61E+00	**6.77E-139**
P5	**Average**	9.97E+02	9.98E+02	9.96E+02	9.97E+02	9.97E+02	9.97E+02	**1.09E+02**
	Std.	2.30E-01	1.93E-01	1.50E-01	**1.27E-01**	1.93E-01	2.98E-01	1.53E+02
P6	**Average**	1.99E+02	2.19E+02	1.92E+02	2.06E+02	1.86E+02	1.98E+02	**1.42E+00**
	Std.	2.13E+00	1.33E+00	2.60E+00	2.12E+00	3.68E+00	1.96E+00	**2.53E-01**
P7	**Average**	2.13E-02	1.08E-03	5.87E-02	1.49E-02	1.092E-02	2.10E-02	**2.47E-04**
	Std.	5.79E-03	4.63E-04	9.94E-03	3.84E-03	5.35E-03	5.76E-03	**2.42E-04**
P8	**Average**	-1.09E+05	-2.08E+04	-9.40E+04	-8.61E+04	-1.08E+05	-1.04E+05	**-2.54E+05**
	Std.	2.46E+04	**1.55E+03**	2.93E+03	3.68E+04	5.99E+03	3.39E+04	2.40E+04
P9	**Average**	3.80E+01	1.82E-13	9.85E+01	1.09E+00	0.00E+00	4.18E+01	**0.00E+00**
	Std.	2.06E+01	5.55E-13	3.49E+01	3.42E+00	0.00E+00	2.25E+01	**0.00E+00**
P10	**Average**	3.59E-07	2.82E-14	4.36E-06	1.61E-08	5.39E-15	3.33E-07	**1.60E-15**
	Std.	5.78E-08	5.29E-15	9.74E-07	4.03E-09	1.60E-15	6.61E-08	**1.45E-15**
P11	**Average**	1.45E-03	1.59E-16	9.34E-03	7.77E-04	0.00E+00	1.49E-03	**0.00E+00**
	Std.	6.02E-03	5.60E-17	1.97E-02	4.25E-03	0.00E+00	8.17E-03	**0.00E+00**
P12	**Average**	7.76E-01	9.52E-01	1.07E+00	8.41E-01	7.13E-01	7.58E-01	**1.64E-03**
	Std.	2.52E-02	1.52E-02	3.18E-01	1.81E-02	2.65E-02	3.47E-02	**3.06E-04**
P13	**Average**	9.69E+01	9.80E+01	1.09E+02	9.55E+01	9.29E+01	9.68E+01	**1.02E+00**
	Std.	1.29E+00	4.52E-01	3.57E+00	7.56E-01	7.13E-01	2.14E+00	**2.00E-01**

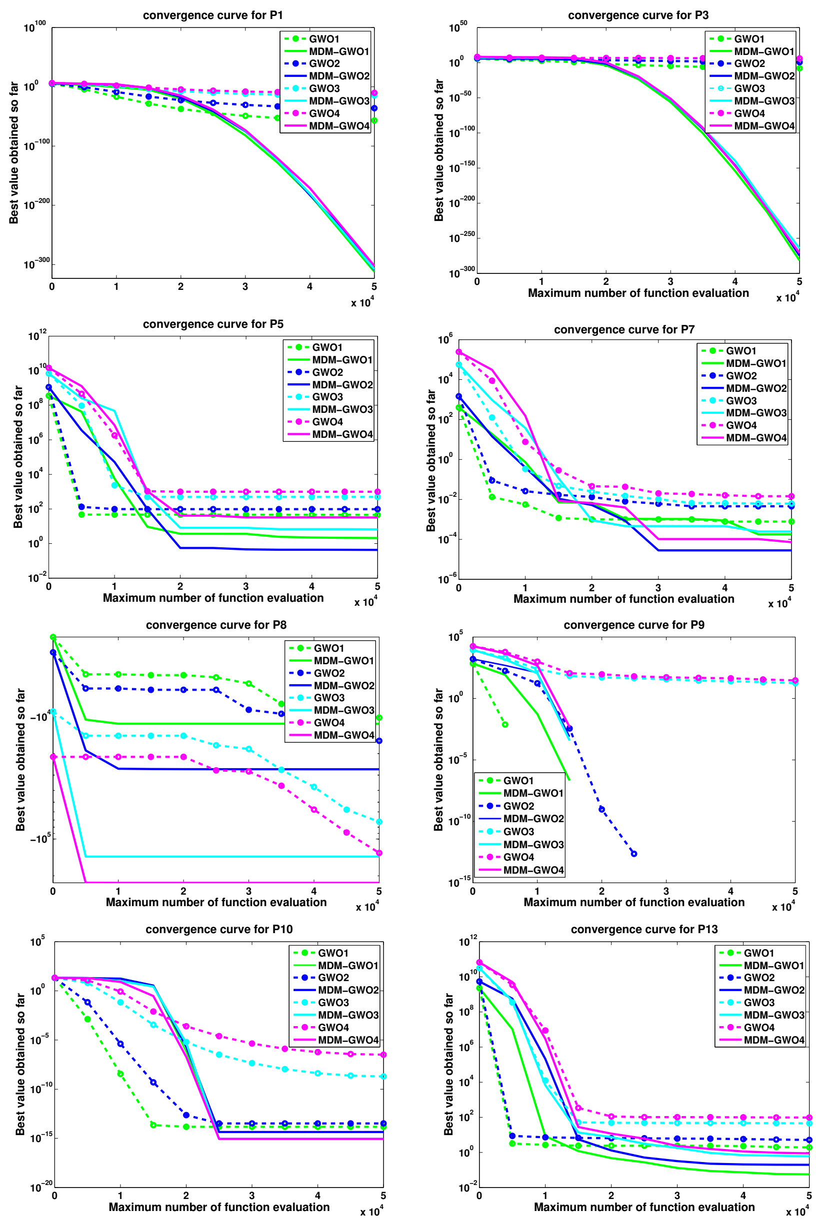

Figure 6: Convergence curves for higher dimensions

MGWO which for 100, 500, and 1000 dimensions test problems, the proposed MDM-GWO is statistically a better performer.

Table 8: Obtained statistical results from Wilcoxon rank-sum test on 23 well-known benchmark problems.

Problem		GWO	OGWO	RW-GWO	MGWO	GWO-XOBL	WF-GWO
P1	**p-value**	3.01E-11	3.02E-11	3.00E-11	3.00E-11	5.20E-02	3.02E-11
	outcome	+	+	+	+	≈	+
P2	**p-value**	3.01E-11	3.02E-11	3.00E-11	3.00E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P3	**p-value**	3.01E-11	3.02E-11	3.00E-11	3.00E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P4	**p-value**	3.01E-11	3.02E-11	3.00E-11	3.00E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P5	**p-value**	3.01E-11	1.21E-10	3.00E-11	3.00E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P6	**p-value**	3.99E-04	3.02E-11	2.71E-02	3.99E-04	9.26E-09	1.11E-06
	outcome	+	+	+	+	+	+
P7	**p-value**	1.07E-07	1.71E-01	1.21E-10	7.70E-04	6.70E-11	5.57E-10
	outcome	+	≈	+	+	+	+
P8	**p-value**	6.77E-05	3.02E-11	1.78E-04	1.25E-04	2.43E-05	1.87E-05
	outcome	+	+	-	+	-	+
P9	**p-value**	2.93E-05	NA	1.66E-11	NA	NA	5.58E-03
	outcome	+	≈	+	≈	≈	+
P10	**p-value**	1.13E-13	1.72E-11	4.17E-13	6.03E-11	3.78E-10	1.55E-13
	outcome	+	+	+	+	+	+
P11	**p-value**	4.19E-02	3.34E-01	4.19E-02	3.34E-01	NA	2.16E-02
	outcome	+	≈	+	≈	≈	+
P12	**p-value**	5.57E-10	3.02E-11	NA	3.02E-11	3.79E-01	3.02E-11
	outcome	+	+	≈	+	≈	+
P13	**p-value**	9.26E-09	3.02E-11	6.63E-01	5.57E-10	3.79E-01	9.76E-10
	outcome	+	+	≈	+	≈	+
P14	**p-value**	4.83E-01	3.02E-11	6.72E-10	1.68E-03	9.21E-05	1.84E-02
	outcome	+	+	+	+	+	+
P15	**p-value**	1.68E-04	3.52E-07	3.57E-06	4.74E-06	1.25E-05	2.84E-04
	outcome	+	-	-	+	+	+
P16	**p-value**	8.10E-10	3.02E-11	4.62E-10	3.99E-04	1.64E-05	2.03E-09
	outcome	-	+	+	+	+	+
P17	**p-value**	1.68E-04	8.15E-11	1.26E-01	4.12E-01	2.58E-01	2.50E-03
	outcome	-	+	≈	≈	≈	-
P18	**p-value**	6.57E-02	8.24E-02	1.68E-04	3.11E-01	3.82E-09	3.87E-01
	outcome	≈	≈	-	≈	-	≈
P19	**p-value**	NA	NA	NA	NA	NA	NA
	outcome	≈	≈	≈	≈	≈	≈
P20	**p-value**	4.08E-05	1.91E-01	2.78E-07	1.34E-05	1.19E-06	3.27E-02
	outcome	-	≈	-	+	-	-
P21	**p-value**	1.63E-02	3.02E-11	2.71E-01	1.33E-01	3.96E-08	2.92E-02
	outcome	+	+	≈	≈	+	+
P22	**p-value**	1.86E-03	3.02E-11	1.04E-04	3.04E-01	1.07E-07	2.15E-06
	outcome	+	+	+	≈	+	+
P23	**p-value**	1.04E-04	3.02E-11	2.60E-05	3.33E-01	1.89E-04	3.03E-03
	outcome	-	+	+	≈	+	+

Table 9: Obtained statistical results from Wilcoxon rank-sum test on scalable benchmark problems with 50 and 100 dimension

Problem		**GWO**	**OGWO**	**RW-GWO**	**MGWO**	**GWO-XOBL**	**Weight-GWO**
For 50 dimension							
P1	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P2	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.92E-02	3.02E-11
	outcome	+	+	+	+	+	+
P3	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P4	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P5	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P6	**p-value**	3.02E-11	3.02E-11	5.57E-10	3.02E-11	6.77E-05	3.02E-11
	outcome	+	+	+	+	+	+
P7	**p-value**	1.96E-10	1.54E-01	3.02E-11	1.20E-08	3.34E-11	4.98E-11
	outcome	+	≈	+	+	+	+
P8	**p-value**	1.55E-09	3.02E-11	7.22E-06	4.99E-09	5.26E-04	1.07E-07
	outcome	+	+	-	+	+	+
P9	**p-value**	2.21E-06	NA	1.21E-12	8.15E-02	NA	2.93E-05
	outcome	+	≈	+	≈	≈	+
P10	**p-value**	2.51E-12	2.94E-13	9.65E-13	4.28E-13	3.94E-12	1.04E-12
	outcome	+	+	+	+	+	+
P11	**p-value**	8.15E-02	NA	2.16E-02	3.34E-01	NA	1.61E-01
	outcome	≈	≈	+	≈	≈	≈
P12	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	8.48E-09	3.02E-11
	outcome	+	+	+	+	+	+
P13	**p-value**	3.02E-11	3.02E-11	4.20E-10	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
For 100 dimension							
P1	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P2	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P3	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P4	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P5	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P6	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P7	**p-value**	3.02E-11	1.03E-06	3.02E-11	1.46E-10	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P8	**p-value**	4.08E-11	3.02E-11	3.92E-02	8.15E-11	1.07E-07	4.50E-11
	outcome	+	+	+	+	+	+
P9	**p-value**	1.68E-08	NA	1.21E-12	3.34E-01	NA	4.76E-08
	outcome	+	≈	+	+	≈	+
P10	**p-value**	3.39E-12	1.94E-12	3.99E-12	2.60E-13	1.87E-08	2.14E-12
	outcome	+	+	+	+	+	+
P11	**p-value**	8.15E-02	3.34E-01	NA	3.34E-01	NA	2.16E-02
	outcome	≈	≈	≈	≈	≈	+
P12	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P13	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+

Table 10: Obtained statistical results from Wilcoxon rank-sum test on scalable benchmark problems on 500 and 1000 dimension

Problem		**GWO**	**OGWO**	**RW-GWO**	**MGWO**	**GWO-XOBL**	**Weight-GWO**
For 500 dimension							
P1	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P2	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P3	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P4	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P5	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P6	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P7	**p-value**	3.02E-11	1.07E-09	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P8	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P9	**p-value**	1.21E-12	1.61E-01	1.21E-12	7.44E-13	NA	1.21E-12
	outcome	+	≈	+	+	≈	+
P10	**p-value**	5.14E-12	2.81E-12	3.15E-12	8.87E-12	1.80E-09	3.15E-12
	outcome	+	+	+	+	+	+
P11	**p-value**	1.19E-12	7.15E-13	1.21E-12	1.60E-13	NA	1.20E-12
	outcome	+	+	+	+	≈	+
P12	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P13	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
For 1000 dimension							
P1	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P2	**p-value**	3.02E-11	3.02E-11	1.21E-12	3.02E-11	1.21E-12	1.21E-12
	outcome	+	+	+	+	+	+
P3	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P4	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P5	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P6	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P7	**p-value**	3.02E-11	8.15E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P8	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P9	**p-value**	1.21E-12	8.14E-02	1.20E-12	1.21E-12	NA	1.21E-12
	outcome	+	+	+	+	≈	+
P10	**p-value**	8.87E-12	5.25E-12	6.32E-12	3.15E-12	5.13E-10	5.14E-12
	outcome	+	+	+	+	+	+
P11	**p-value**	1.21E-12	4.46E-13	1.21E-12	1.21E-12	NA	1.21E-12
	outcome	+	+	+	+	≈	+
P12	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+
P13	**p-value**	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+

Table 11: Obtained average ranking of the algorithms by the Friedman test

30 Dimensional		50 Dimensional		100 Dimensional		500 Dimensional		1000 Dimensional	
MDM-GWO	**2.28E+00**	**MDM-GWO**	**1.38E+00**	**MDM-GWO**	**1.15E+00**	**MDM-GWO**	**1.08E+00**	**MDM-GWO**	**1.15E+00**
GWO-XOBL	2.63E+00	**GWO-XOBL**	2.38E+00	**GWO-XOBL**	2.46E+00	**GWO-XOBL**	2.08E+00	**GWO-XOBL**	2.08E+00
RW-GWO	3.72E+00	**MGWO**	4.19E+00	**MGWO**	4.31E+00	**OGWO**	4.76E+00	**GWO**	4.54E+00
GWO	3.98E+00	**OGWO**	4.57E+00	**OGWO**	4.77E+00	**MGWO**	4.85E+00	**MGWO**	4.76E+00
OGWO	4.63E+00	**RW-GWO**	5.07E+00	**RW-GWO**	4.77E+00	**GWO**	4.77E+00	**WF-GWO**	4.77E+00
WF-GWO	4.48E+00	**WF-GWO**	5.15E+00	**GWO**	4.92E+00	**WF-GWO**	4.92E+00	**OGWO**	4.92E+00
MGWO	6.28E+00	**GWO**	5.23E+00	**WF-GWO**	5.62E+00	**RW-GWO**	5.54E+00	**RW-GWO**	5.77E+00

Table 12: Obtained statistical results from the post-hoc test

30 Dimensional			50 Dimensional			100 Dimensional			500 Dimensional			1000 Dimensional		
Algorithm	**Adjusted p value**	**0.05/rank**	**Algorithm**	**Adjusted p value**	**0.05/rank**	**Algorithm**	**Adjusted p value**	**0.05/rank**	**Algorithm**	**Adjusted p value**	**0.05/rank**	**Algorithm**	**Adjusted p value**	**0.05/rank**
GWO-XOBL	2.98E-01	5.00E-02	**GWO-XOBL**	3.65E-01	5.00E-02	**RW-GWO**	4.17E-02	5.00E-02	**GWO-XOBL**	2.42E-02	5.00E-02	**GWO-XOBL**	3.00E-03	5.00E-02
RW-GWO	2.21E-01	2.50E-02	**RW-GWO**	1.22E-02	2.50E-02	**GWO-XOBL**	1.89E-02	2.50E-02	**WF-GWO**	4.35E-03	2.50E-02	**WF-GWO**	5.88E-07	2.50E-02
GWO	1.09E-02	1.67E-02	**WF-GWO**	1.44E-02	1.67E-02	**WF-GWO**	1.92E-02	1.67E-02	**GWO**	7.00E-03	1.67E-02	**GWO**	5.18E-07	1.67E-02
OGWO	6.00E-03	1.25E-02	**GWO**	8.96E-02	1.25E-02	**GWO**	3.02E-02	1.25E-02	**MGWO**	6.43E-04	1.25E-02	**MGWO**	1.72E-07	1.25E-02
WF-GWO	1.00E-04	1.00E-02	**MGWO**	7.06E-03	1.00E-02	**MGWO**	2.67E-03	1.00E-02	**RW-GWO**	6.20E-06	1.00E-02	**RW-GWO**	9.33E-10	1.00E-02
MGWO	1.08E-06	8.33E-03	**OGWO**	7.80E-06	8.33E-03	**OGWO**	1.61E-08	8.33E-03	**OGWO**	3.41E-08	8.33E-03	**OGWO**	5.77E-12	8.33E-03

4.6. Comparison with metaheuristic algorithms

In this subsection, the proposed MDM-GWO is compared with metaheuristic algorithms such as PSO (Kennedy and Eberhart, 1995), BBO (Simon, 2008), SCA (Mirjalili, 2016), GSA (Rashedi et al., 2009), Salp Swarm Algorithm (SSA) (Mirjalili et al., 2017), Cockoo search optimization algorithm (CS), Harris hawks optimization algorithm (HHO) and Covariance matrix adaptation evolution strategy (CMA-ES) (Hansen, 2006). The parameters of these algorithms are set as follows: the population size of PSO, BBO, SCA, GSA, SSA, CS, HHO, and CMA-ES is 50, the maximum function evaluations for PSO, BBO, SCA, GSA, SSA, CS, HHO and CMA-ES are set to 5×10^4. In these experiments, each algorithm is independently executed 30 times on each problem. The experimental results are presented in Table 13. For the unimodal problems P1-P7, the proposed MDM-GWO achieves the best solutions for problems P1, P2, P3, P4 compared to other metaheuristic algorithms and P5, P7, and P8, HHO algorithm is better than other metaheuristic algorithms. For P6, GSA achieves global optimal solution. For the multimodal problems (P8-P13), MDM-GWO has better performance in the case of P9, and P11. MDM-GWO achieves global optimal solution in P9 and P11, and also HHO algorithm achieves global optimal solution. For problem P8, BBO and HHO achieve global optimal solution (-1.26×10^4), and HHO performs significantly better in problem P10 and in problem P12, and P13 CMA-ES performs better. For multimodal problems with fixed dimensions (P14-P23), MDM-GWO is better than considered metaheuristic algorithms in the case of P14, P19, P21, P22, and P23. For the problem P20, GSA is a better optimizer, and PSO better performance in P15. To verify statistical significance of results for MDM-GWO over considered metaheuristic algorithms, Wilcoxon rank-sum test is performed. To verify the performance of MDM-GWO, significance level is chosen as 5%, and the statistical results are shown in Table 14. Moreover, Table 15 shows the average ranking produced by the Friedman test. The obtained p-value of the Friedman test is 3.44E-07. Furthermore, the post-hoc tests (Holms and Hochberg) has also been applied, and the results of the Holms and Hochberg tests are presented in Table 16. From the results, it is clear that the proposed MDM-GWO is performing better than SCA, BBO, GSA, CS, and CMA-ES. From experiments and overall analysis, it can be concluded that the proposed MDM-GWO is a better optimizer.

Table 13: Comparison of results obtained by MDM-GWO with selected metaheuristic algorithms on 23 well-known benchmark problems

Problem		PSO	BBO	SCA	GSA	SSA	CS	HHO	CMA-ES	MDMGWO
P1	**Average**	2.86E-04	5.75E+00	2.52E-02	1.99E-17	8.33E-09	2.26E-05	2.52E-193	1.63E-54	**0.00E+00**
	Std.	3.68E-04	2.24E+00	1.32E-01	4.54E-18	1.60E-09	1.13E-05	0.00E+00	2.41E-54	**0.00E+00**
P2	**Average**	7.10E-04	8.44E-01	1.03E-05	2.42E-08	4.83E-01	5.72E-02	5.56E-100	1.91E-25	**9.77E-157**
	Std.	5.38E-04	1.50E-01	1.94E-05	3.69E-09	5.98E-01	9.52E-03	4.56E-14	1.76E-25	**3.21E-156**
P3	**Average**	1.67E+03	9.27E+03	2.72E+03	2.67E+02	3.74E+01	1.22E+00	2.78E-163	3.63E-44	**3.21E-277**
	Std.	9.68E+02	2.69E+03	2.65E+03	1.02E+02	2.14E+01	2.62E-01	0.00E+00	7.13E-44	**0.00E+00**
P4	**Average**	1.04E+01	6.34E+00	1.23E+01	1.93E-02	3.94E+00	1.74E-01	7.51E-96	1.83E-20	**9.63E-149**
	Std.	2.37E+00	1.17E+00	8.95E+00	1.06E-01	1.87E+00	3.60E-02	4.11E-95	1.95E-20	**3.32E-148**
P5	**Average**	1.20E+02	3.39E+02	1.48E+02	2.61E+01	1.03E+02	2.62E+01	**1.45E-03**	5.03E+00	1.24E+00
	Std.	1.30E+02	1.66E+02	5.54E+02	2.47E-01	1.27E+02	6.44E-01	**2.15E-03**	8.97E-01	1.24E+00
P6	**Average**	3.79E-04	5.61E+00	4.26E+00	**0.00E+00**	8.91E-09	3.09E-05	1.07E-05	2.75E-30	9.34E-03
	Std.	4.67E-04	1.89E+00	3.83E-01	**0.00E+00**	1.87E-09	1.34E-05	1.72E-05	7.65E-31	2.59E-03
P7	**Average**	5.62E-02	2.63E-02	1.86E-02	2.12E-02	5.60E-02	5.61E-02	**5.23E-05**	5.19E-01	2.34E-04
	Std.	2.10E-02	1.06E-02	1.35E-02	1.01E-02	1.97E-02	2.03E-02	**6.54E-05**	2.90E-01	2.19E-04
P8	**Average**	-9.34E+03	**-1.26E+04**	-4.06E+03	-2.73E+03	-7.73E+03	-1.18E+02	**-1.26E+04**	-1.18E+02	-7.89E+03
	Std.	4.72E+02	5.44E+00	2.48E+02	4.26E+02	6.93E+02	**1.06E-04**	4.00E+01	4.57E-14	1.43E+03
P9	**Average**	2.63E+01	2.70E+00	1.06E+01	1.56E+01	5.58E+01	8.90E+01	**0.00E+00**	1.41E+01	**0.00E+00**
	Std.	7.17E+00	1.00E+00	1.81E+01	3.23E+00	1.81E+01	1.09E+01	**0.00E+00**	3.05E+00	**0.00E+00**
P10	**Average**	7.24E-03	1.17E+00	1.25E+01	3.47E-09	1.70E+00	7.70E-02	**8.88E-16**	5.27E-15	1.72E-15
	Std.	5.76E-03	2.50E-01	9.78E+00	5.00E-10	8.11E-01	5.81E-02	**0.00E+00**	1.53E-15	1.53E-15
P11	**Average**	1.40E-02	1.05E+00	1.73E-01	4.17E+00	9.30E-03	9.02E-05	**0.00E+00**	**0.00E+00**	**0.00E+00**
	Std.	1.09E-02	1.80E-02	2.24E-01	1.95E+00	1.05E-02	1.02E-04	**0.00E+00**	**0.00E+00**	**0.00E+00**
P12	**Average**	1.39E-01	4.62E-02	1.06E+00	3.46E-02	3.81E+00	1.03E-03	7.54E-07	**1.07E-31**	1.56E-03
	Std.	2.02E-01	4.72E-02	1.75E+00	4.97E-02	2.44E+00	1.06E-03	1.17E-06	**2.91E-32**	4.79E-04
P13	**Average**	3.02E-02	2.69E-01	2.43E+02	3.66E-04	8.70E-03	7.46E-04	8.42E-06	**1.55E-30**	3.33E-02
	Std.	6.46E-02	8.45E-02	1.25E+03	2.00E-03	1.96E-02	4.52E-04	1.34E-05	**4.25E-31**	2.04E-02
P14	**Average**	**9.98E-01**	1.00E+00	1.26E+00	3.48E+00	**9.98E-01**	1.27E+01	**9.98E-01**	1.27E+01	**9.98E-01**
	Std.	**0.00E+00**	9.92E-03	6.84E-01	2.05E+00	1.92E-16	2.47E-15	9.28E-11	1.58E-13	1.99E-10
P15	**Average**	4.61E-04	7.23E-03	8.25E-04	2.20E-03	1.50E-03	**3.07E-04**	3.55E-04	**3.07E-04**	8.96E-04
	Std.	2.33E-04	8.35E-03	3.38E-04	1.10E-03	3.60E-03	2.11E-08	1.84E-04	**1.97E-19**	3.29E-04
P16	**Average**	-1.03E+00	-1.03E+00	-1.03E+00	**-1.03E+00**	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	**-1.03E+00**
	Std.	6.71E-16	1.34E-03	1.66E-05	5.53E-16	9.25E-15	6.78E-16	4.69E-13	**4.88E-16**	2.55E-08
P17	**Average**	**3.98E-01**	3.99E-01	3.98E-01	**3.98E-01**	3.98E-01	3.98E-01	3.98E-01	**3.98E-01**	3.98E-01
	Std.	**0.00E+00**	2.06E-03	3.67E-04	**0.00E+00**	3.48E-15	**0.00E+00**	1.29E-07	**0.00E+00**	3.43E-07
P18	**Average**	**3.00E+00**	4.84E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Std.	**6.70E-16**	6.87E+00	1.99E-05	2.84E-15	5.51E-14	1.01E-15	1.37E-10	8.47E-15	2.54E-06
P19	**Average**	-1.19E-02	-2.81E-01	-3.00E-01	**-3.86E+00**	-3.01E-01	**-3.86E+00**	**-3.86E+00**	-3.77E+00	-3.00E-01
	Std.	2.10E-02	1.23E-02	2.26E-16	2.49E-15	2.26E-16	2.71E-15	5.01E-04	1.35E-15	**2.26E-16**
P20	**Average**	-2.11E+00	-3.26E+00	-3.02E+00	**-3.32E+00**	-3.22E+00	-3.32E+00	-3.17E+00	-3.31E+00	-3.22E+00
	Std.	3.96E-01	6.05E-02	1.42E-01	**1.36E-15**	4.55E-02	1.59E-09	8.13E-02	3.76E-02	6.10E-02
P21	**Average**	-1.32E+00	-5.69E+00	-3.33E+00	-6.22E+00	-9.40E+00	**-1.02E+01**	-5.22E+00	-5.06E+00	**-1.02E+01**
	Std.	8.94E-01	3.49E+00	2.03E+00	3.54E+00	1.99E+00	5.71E-15	9.28E-01	**5.13E-16**	3.32E-03
P22	**Average**	-1.43E+00	-5.69E+00	-4.68E+00	-1.04E+01	-9.97E+00	**-1.04E+01**	-5.44E+00	-5.09E+00	**-1.04E+01**
	Std.	8.23E-01	3.38E+00	1.63E+00	**5.71E-16**	1.67E+00	6.46E-15	1.34E+00	7.25E-16	7.15E-03
P23	**Average**	-1.39E+00	-6.39E+00	-5.47E+00	-1.05E+01	-1.02E+01	**-1.05E+01**	-5.31E+00	-5.13E+00	**-1.05E+01**
	Std.	5.20E-01	3.64E+00	1.72E+00	1.71E-15	1.37E+00	1.62E-15	9.80E-01	**5.13E-16**	6.32E-03

Table 14: Obtained statistical results from Wilcoxon rank-sum test on 23 well-known benchmark problems

Problem		PSO	BBO	SCA	GSA	SSA	CS	HHO	CMA-ES
P1	**p-value**	3.01E-11	3.01E-11	3.01E-11	3.01E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+	+	+
P2	**p-value**	3.01E-11	3.01E-11	3.01E-11	3.01E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+	+	+
P3	**p-value**	3.01E-11	3.01E-11	3.01E-11	3.01E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+	+	+
P4	**p-value**	3.01E-11	3.01E-11	3.01E-11	3.01E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
	outcome	+	+	+	+	+	+	+	+
P5	**p-value**	3.01E-11	3.01E-11	3.01E-11	3.01E-11	4.08E-11	NA	3.02E-11	3.69E-11
	outcome	+	+	+	+	+	NA	-	+
P6	**p-value**	3.01E-11	3.01E-11	3.01E-11	1.21E-12	3.02E-11	9.51E-06	1.70E-08	3.02E-11
	outcome	+	+	+	-	+	+	-	-
P7	**p-value**	3.01E-11	3.01E-11	3.01E-11	3.02E-11	3.02E-11	3.02E-11	1.61E-10	3.02E-11
	outcome	+	+	+	+	+	+	-	+
P8	**p-value**	1.60E-07	3.02E-11	3.02E-11	3.02E-11	5.79E-01	3.02E-11	3.02E-11	4.08E-12
	outcome	+	-	+	+	≈	-	-	+
P9	**p-value**	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.44E-11	3.13E-04	1.17E-12
	outcome	+	+	+	+	+	+	+	+
P10	**p-value**	3.15E-12	5.14E-12	4.08E-12	1.60E-07	8.87E-12	1.21E-12	1.69E-14	4.51E-11
	outcome	+	+	+	+	+	+	-	+
P11	**p-value**	1.21E-12	1.21E-12	1.21E-12	1.21E-12	1.21E-12	4.28E-06	2.16E-02	NA
	outcome	+	+	+	+	+	+	+	≈
P12	**p-value**	1.07E-07	3.02E-11	3.02E-11	3.02E-11	3.02E-11	5.57E-10	9.92E-11	3.02E-11
	outcome	+	+	+	+	+	+	-	-
P13	**p-value**	2.38E-03	3.02E-11	3.02E-11	3.02E-11	5.53E-08	5.57E-10	3.34E-11	3.02E-11
	outcome	+	+	+	-	+	+	-	-
P14	**p-value**	1.21E-12	4.50E-11	3.02E-11	3.02E-11	7.57E-12	2.37E-10	1.78E-04	2.72E-11
	outcome	+	+	+	+	+	+	+	+
P15	**p-value**	1.73E-06	1.11E-06	8.07E-01	8.88E-01	8.88E-01	2.00E-06	7.96E-03	8.39E-09
	outcome	-	+	≈	≈	≈	-	-	-
P16	**p-value**	1.72E-12	3.02E-11	3.02E-11	2.98E-11	2.98E-11	1.21E-12	2.91E-11	3.15E-12
	outcome	-	+	+	-	-	-	-	-
P17	**p-value**	1.21E-12	4.08E-11	3.02E-11	1.21E-12	2.37E-11	1.21E-12	6.01E-08	1.21E-12
	outcome	-	+	+	-	-	-	+	-
P18	**p-value**	6.47E-12	3.02E-11	7.70E-04	6.47E-12	3.02E-11	2.46E-11	4.50E-11	2.59E-11
	outcome	-	+	+	-	-	-	-	-
P19	**p-value**	1.21E-12	1.21E-12	NA	1.21E-12	NA	1.69E-14	NA	NA
	outcome	+	+	≈	-	≈	+	≈	≈
P20	**p-value**	3.02E-11	6.28E-06	3.20E-09	1.10E-12	4.98E-04	3.02E-11	6.77E-05	5.29E-07
	outcome	+	-	+	-	+	-	+	-
P21	**p-value**	3.02E-11	8.99E-11	3.02E-11	8.99E-11	1.11E-06	1.48E-11	3.02E-11	6.43E-12
	outcome	+	+	+	+	+	-	+	+
P22	**p-value**	3.02E-11	4.98E-11	3.02E-11	3.02E-11	8.48E-09	3.16E-12	3.02E-11	1.48E-11
	outcome	+	+	+	-	+	-	+	+
P23	**p-value**	3.02E-11	3.47E-10	3.02E-11	3.02E-11	8.48E-09	1.34E-11	3.02E-11	1.48E-11
	outcome	+	+	+	-	+	-	+	+

Table 15: Obtained average ranking of the algorithms by the Friedman test

Algorithm	Ranking
MDM-GWO	3.28E+00
HHO	3.30E+00
CMA-ES	3.91E+00
GSA	4.28E+00
CS	4.96E+00
SSA	5.35E+00
PSO	6.04E+00
BBO	6.83E+00
SCA	6.93E+00

Table 16: Obtained statistical results from the post-hoc test

Algorithm	Adjusted p-value	0.05/rank
SSA	4.13E-01	5.00E-02
PSO	1.17E-01	2.50E-02
SCA	1.40E-02	1.67E-02
BBO	4.72E-06	1.25E-02
GSA	3.26E-11	1.00E-02
CS	0.00E+00	8.33E-03
CMA-ES	0.00E+00	7.14E-03

4.7. MDM-GWO for engineering design problems

In this section, MDM-GWO is applied to solve engineering design problems. These are unconstrained and constrained-type problems in nature. To deal with the constraints of optimization problems, we used the adaptive penalty function approach of constraint handling (Deb, 2000) for all experiments. The objective function $\zeta(x)$ in the constrained optimization problem corresponding to the solution x obtained by the proposed MDM-GWO is written in the following way:

$$\zeta(x) = \begin{cases} f(x) & \text{if } x \in X \\ f(x) + \underbrace{\sum_{i=1}^{m} \lambda_i \times U_i(x)^{\alpha_1} + \sum_{j=m+1}^{p} \eta_j \times V_j(x)^{\alpha_1}}_{constraint \quad violation} & \text{if } x \notin X \end{cases} \tag{20}$$

where, $x = (x_1, x_2, \cdots, x_n) \in X \subset \mathbb{R}^n$. λ_i and η_j are two non-negative penalty coefficients. m and p are the numbers of inequality constraints and equality constraints and $U_i(x)$ and $V_j(x)$ are represent inequality constraints and equality constraints for the problem. Parameter α_1 and α_2 take value 2.

4.7.1. Gear train design

This problem is an unconstrained type, and it was proposed by Sandgren (Sandgren, 1990). The goal of this problem is to minimize the cost of gear ratio. Figure 7 shows the gear design problem. This problem has four positive integer variables, namely x_1, x_2, x_3, and x_4 which lie in the boundary $[12, 60]$. These variables indicate the number of teeth.

Mathematically, the Gear train design problem can be formulated as follows:

$$min \quad f_1(X) = \left(\frac{1}{6.931} - \frac{x_2 x_3}{x_1 x_4}\right)^2 \tag{21}$$

where, $X = (x_1, x_2, x_3, x_4)$

$$\text{s.t. } 12 \leq x_1, x_2, x_3, x_4 \leq 60.$$

Table 17 shows the comparison of the results obtained using MDM-GWO, conventional GWO and other variants of GWO MGWO, OGWO, RW-GWO, GWO-XOBL, WF-GWO algorithms. To determine the solution of this problem, the population size and function evaluations are fixed to be 30 and 15000. The results in Table 17 show that MDM-GWO is not worse than any other algorithm for solving the Gear train design problem and the optimal solution is corresponding to the optimal cost $2.7009E-12$ [49 , 16 , 19 , 43].

Table 17: Optimization results on gear train design problem

Algorithm	Decision variable				Optimal cost
	x_1	x_2	x_3	x_4	
MGWO	49	19	16	43	2.70E-12
OBGWO	34	13	20	53	2.31E-11
RW-GWO	51	15	26	53	2.31E-11
GWO	43	16	19	49	2.70E-12
GWO-XOBL	37	16	16	48	1.83E-08
WF-GWO	43	19	16	49	2.70E-12
MDM-GWO	**49**	**16**	**19**	**43**	**2.70E-12**

4.7.2. Speed reducer design

The speed reducer design problem has been used as a benchmark structural design problem. The goal of this problem is to minimize the total weight subject to some constraints (Mezura-Montes and Coello, 2005). Figure 8 illustrates the speed reducer design problem. It consists of seven

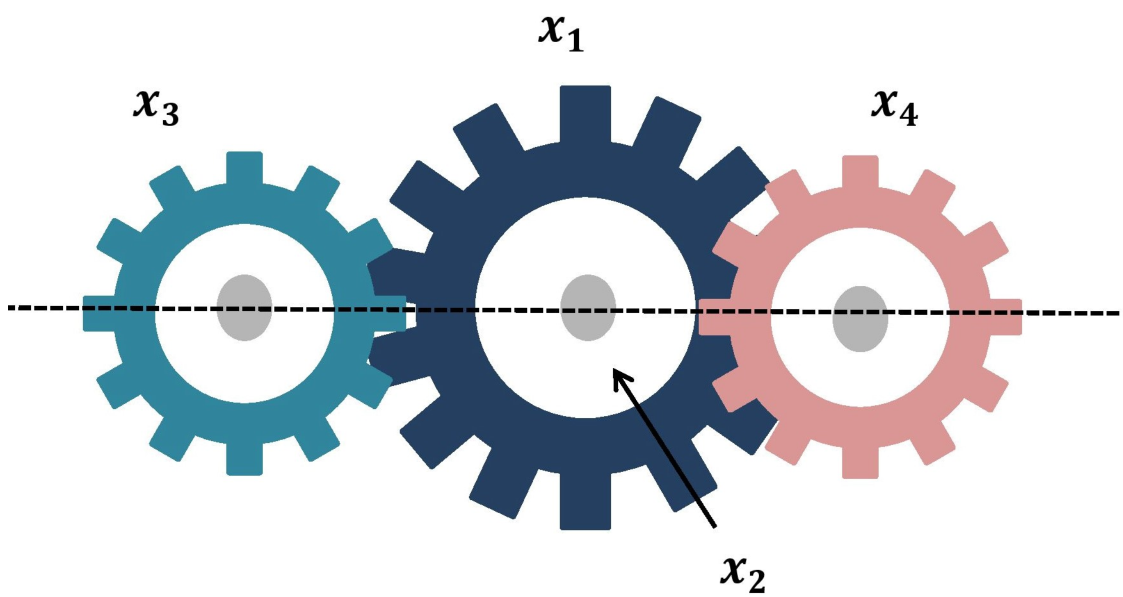

Figure 7: Gear train design problem

decision variables: b, m, z, l_1, l_2, d_1 and d_2. These variables defined as face width, the module of teeth, number of teeth on pinion, length of shaft 1 between bearings, length of shaft 2 between bearings, diameter of shaft, and diameter of shaft and are denoted by x_1, x_2, x_3, x_4, x_5, x_6 and x_7, respectively.

Mathematically, this problem can be formulated as follows:

$$\begin{aligned} \min \qquad f_2(X) = {} & 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) \\ & +7.4770(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2) \end{aligned} \tag{22}$$

$$s.t. \quad \frac{27}{x_1 x_2^2 x_3} - 1 \leq 0 \tag{23}$$

$$\frac{397.5}{x_1 x_2^2 x_3^2} - 1 \leq 0 \tag{24}$$

$$\frac{1.93 x_4^3}{x_2 x_3 x_6^4} - 1 \leq 0 \tag{25}$$

$$\frac{1.93 x_5^3}{x_2 x_3 x_7^4} - 1 \leq 0 \tag{26}$$

$$\frac{\sqrt{1.69 \times 10^6 + (\frac{745 x_4}{x_2 x_3})^2}}{110 x_6^3} - 1 \leq 0 \tag{27}$$

$$\frac{\sqrt{157.5 \times 10^6 + (\frac{745 x_4}{x_2 x_3})^2}}{85 x_7^3} - 1 \leq 0 \tag{28}$$

$$\frac{x_2 x_3}{40} - 1 \leq 0 \tag{29}$$

$$\frac{x_1}{12 x_2} - 1 \leq 0 \tag{30}$$

$$\frac{5 x_2}{x_1} - 1 \leq 0 \tag{31}$$

where $X = (x_1, x_2, x_3, x_4, x_5, x_6, x_7) \in \mathbb{R}^7$

The optimal results obtained by the proposed MDM-GWO, the conventional GWO, and other algorithms MGWO, OGWO, RW-GWO, GWO-XOBL, WF-GWO are shown in Table 18. In this table, optimal value is 2997.0683 corresponding to optimal weight [3.500645, 0.700000, 17.000000, 7.309897, 7.808962, 3.350328, 5.286125]. It can be seen that MDM-GWO has a lower cost than that of RW-GWO.

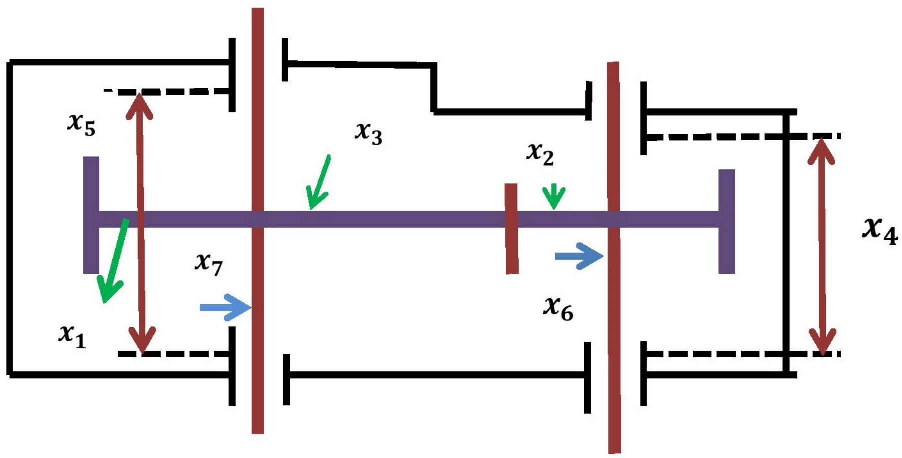

Figure 8: Speed reducer design problem

Table 18: Optimization results on Speed reducer design problem

Algorithm	Decision variable							Optimal cost
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
MGWO	3.499820	0.700000	17.013068E	7.4166313	7.838926E	3.3584894	5.2873136	3002.9519
OBGWO	3.600000	0.700000	17.000000	7.300000	7.800000	3.364043	5.289516	3040.9482
RW-GWO	**3.500645**	**0.700000**	**17.000000**	**7.309897**	**7.808962**	**3.350328**	**5.286125**	**2997.0683**
GWO	3.600000	0.800000	28.000000	8.153447	7.800000	2.900000	5.000000	3038.0094
GWO-XOBL	3.502246	0.700000	17.000000	7.300000	7.800000	3.353244	5.288644	2999.2508
WF-GWO	3.502501	0.700000	17.000000	7.437258	7.806525	3.350118	5.286136	2999.3043
MDM-GWO	3.501335	0.7000000	17.000000	7.300000	7.800000	3.357801	5.285996	2999.1348

4.7.3. Three-bar truss design

A three-bar truss design problem was introduced by (Yang, 2013), where the volume of a bar truss is minimized subject to stress constraints on each of the truss members. The details of the three-bar truss design problem are shown in Figure 9. This problem contains two parameters, three constraints, and one objective.

Mathematically, the Three-bar truss design problem can be formulated as follows:

$$
\begin{aligned}
\min \quad & f_2(x_1, x_2) = L \times (2\sqrt{2}x_1 + x_2) && (32)\\
s.t. \quad & \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1y_2} P - \sigma \leq 0 && (33)\\
& \frac{x_2}{\sqrt{2}x_1^2 + xy_1x_2} P - \sigma \leq 0 && (34)\\
& \frac{1}{x_1 + \sqrt{2}x_2} P - \sigma \leq 0 && (35)\\
& && (36)
\end{aligned}
$$

where $0 \leq x_1, x_2 \leq 1$
$L = 100cm, \sigma, P = 2KN/cm^2$

Some researchers (Ray and Saini, 2001; Gupta et al., 2020; Fan et al., 2021b) have solved this problem using various methods. In this paper, the MDM-GWO is run 30 times independently and the number of function evaluations are fixed (same as (Gandomi et al., 2013)). The obtained results are shown in Table 19 and are compared with MGWO, OGWO, RW-GWO, GWO-XOBL, WF-GWO. As can be seen from Table 19, MDM-GWO is superior to all the other algorithms and obtains the best solution [0.788788, 0.407928] corresponding to the optimal cost 263.895812.

Table 19: Optimization results on three-bar truss design problem

Algorithm	**Decision variable**		**Optimal cost**
	x_1	x_2	
MGWO	0.789208	0.406743	263.896160
OBGWO	0.788254	0.409462	263.898253
RW-GWO	0.788383	0.409071	263.895871
GWO	0.925638	0.162327	263.905187
GWO-XOBL	0.788841	0.407781	263.896203
WF-GWO	0.788460	0.408852	263.895962
MDM-GWO	**0.788788**	**0.407928**	**263.895812**

4.7.4. Pressure vessel design

This problem has four decision variables namely the thickness of the shell (T_s), head (T_h), inner radius (R) and range of cross-section minus head (L) (Gandomi et al., 2013). T_h and T_s are integer multiples of 0.0625. The goal

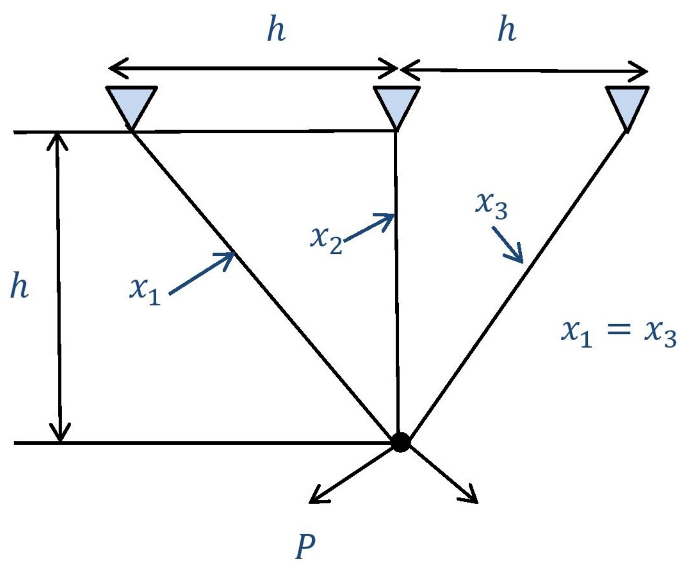

Figure 9: Three-bar truss design problem

of this problem is to find the least fabrication cost to obtain a design for pressure vessel. T_h, T_s, R and L are defined by x_1, x_2, x_3 and x_4. This problem is illustrated in Figure 10.

The mathematical formation for this problem is expressed as follows:

$$Min \quad f_4(X) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3 \quad (37)$$

where, $X = (x_1, x_2, x_3, x_4) \in \mathbb{R}^4$

$$
\begin{aligned}
s.t. \qquad & -x_1 + 0.0193x_3 \leq 0 && (38)\\
& -x_2 + 0.00954y_3 \leq 0 && (39)\\
& -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \leq 0 && (40)\\
& x_4 - 240 \leq 0 && (41)
\end{aligned}
$$

$0 \leq x_1, x_2 \leq 99$
$10 \leq x_3, x_4 \leq 200$

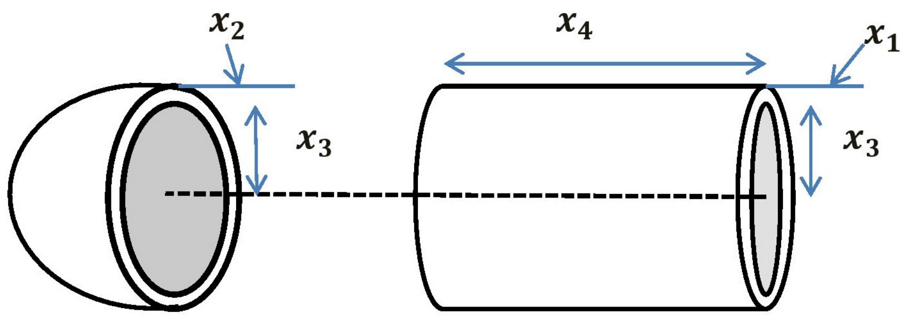

Figure 10: Pressure vessel design problem

Table 20 shows the list of the optimal solutions to this problem obtained by MDM-GWO, conventional GWO, MGWO, OGWO, RW-GWO, GWO-XOBL, and WF-GWO. For a fair comparison, the results are obtained using 2×10^4 number of function evaluations. From the Table 20, MDM-GWO obtains a optimal value 5909.3999 corresponding to optimal solution [0.782604, 0.385883, 40.466110, 198.2206206]. As can be seen from the results that MDM-GWO is outperformed all other metaheuristic algorithms to solve the pressure design problem.

Table 20: Optimization results on Pressure vessel design problem

Algorithm	**Decision variable**				**Optimal cost**
	x_1	x_2	x_3	x_4	
MGWO	0.783073	0.400581	40.465902	198.707525	5963.4353
OBGWO	0.785255	0.389159	40.661777	195.847138	5915.9859
RW-GWO	0.800709	0.401092	41.496345	184.348687	5945.3566
GWO	0.381467	1.421170	10.000000	181.733831	8872.24681
GWO-XOBL	0.786464	0.390236	40.71381	194.753886	5912.4872
WF-GWO	0.798453	0.394658	41.308956	186.678317	5929.0637
MDM-GWO	**0.782604**	**0.385883**	**40.466110**	**198.220620**	**5909.3999**

In overall summary, the results on the engineering design problems show that the proposed MDM-GWO is performed better than that of the conventional GWO and other variants of GWO namely MGWO, OGWO, RW-GWO, GWO-XOBL, and WF-GWO. Hence, it can be concluded that MDM-GWO not only has increased global search ability but also helps in preventing the search stuck to local optima to effectively solve engineering design problems.

5. Conclusions

In this paper, a new variant of conventional GWO called MDM-GWO is proposed by combining the four different strategies, namely a new update search mechanism, modified control parameter, mutation-driven scheme, and greedy approach for better balance between exploration and exploitation while maintaining the higher convergence speed. The performance of the proposed MDM-GWO is measured on 23 benchmark test problems with 5 different complexity levels. The obtained experimental results are compared with other variants of GWO and other popular meta-heuristics. Statistical analysis and diversity analysis have been carried out and showed that the employed strategies are successful to upgrade the performance of the GWO. Moreover, to visualize the performance of the MDM-GWO on real-life optimization problems, four engineering design problems are considered to solve. The results obtained by the proposed MDM-GWO for the engineering optimization problems are superior to the conventional GWO and other optimization methods. Overall, it can be concluded that the proposed MDM-GWO has the potential to be used for solving real-world optimization problems. MDM-GWO can also be solved multi-objective, discrete optimization problems and various application.

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