# Artificial bee colony algorithm with global and local neighborhoods 

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#### Abstract

Artificial Bee Colony ( ABC ) is a well known population based efficient algorithm for global optimization. Though, ABC is a competitive algorithm as compared to many other optimization techniques, the drawbacks like preference on exploration at the cost of exploitation and slow convergence are also associated with it. In this article, basic ABC algorithm is studied by modifying its position update equation using the differential evolution with global and local neighborhoods like concept of food sources' neighborhoods. Neighborhood of each colony member includes $10 \%$ members from the whole colony based on the index-graph of solution vectors. The proposed $A B C$ is named as ABC with Global and Local Neighborhoods (ABCGLN) which concentrates to set a trade off between the exploration and exploitation and therefore increases the convergence rate of ABC . To validate the performance of proposed algorithm, ABCGLN is tested over 24 benchmark optimization functions and compared with standard ABC as well as its recent popular variants namely, Gbest guided ABC , Best-So-Far ABC and Modified ABC . Intensive statistical analyses of the results shows that ABCGLN is


[^0]significantly better and takes on an average half number of function evaluations as compared to other considered algorithms.

Keywords Artificial bee colony - Optimization • Exploration-exploitation • Swarm intelligence

## 1 Introduction

Swarm Intelligence is one of the recent outcome of the research in the field of Nature inspired algorithms. Collaborative trial and error method is the main concept behind the swarm intelligence which enables the algorithmic procedure to find the solution. Researchers are analyzing such collaboration among the social insects while searching food for them and creating the intelligent structures known as Swarm Intelligence. Spider monkey optimization (SMO) (Bansal et al. 2013), Ant colony optimization (ACO) (Dorigo and Di Caro 1999), Particle swarm optimization (PSO) (Kennedy and Eberhart 1995), Bacterial foraging optimization (BFO) (Passino 2002) are some examples of swarm intelligence based techniques. The work presented in the articles (Dorigo and Di Caro 1999; Kennedy and Eberhart 1995; Price et al. 2005; Vesterstrom and Thomsen 2004) proved its efficiency and potential to deal with non linear, non convex and discrete optimization problems.

Karaboga (2005) contributed the recent addition to this category known as Artificial bee colony (ABC) optimization algorithm. ABC is a simple but effective algorithm for both continuous and discrete optimization problems. The ABC algorithm has emerged as one of the popular tool in the machine intelligence and has successfully been tested
on almost all domains of science and engineering like electronics engineering (Kavian et al. 2012; Chidambaram and Lopes 2009), electrical engineering (Jones and Bouffet 2008; Nayak et al. 2009; Sulaiman et al. 2012), computer science engineering (Lei et al. 2010; Karaboga and Cetinkaya 2011; Lam et al. 2012), mechanical engineering (Pawar et al. 2008; Xu and Duan 2010; Banharnsakun et al. 2012), civil engineering (Li et al. 2011; Mandal et al. 2012; Akay and Karaboga 2012), medical pattern classification and clustering problems (Akay et al. 2008) and mathematical graph problems (Xing et al. 2007; Singh 2009; Yeh and Hsieh 2011). Many of the recent modifications and applications of ABC algorithm can be studied in Bansal et al. (2013).

The ABC algorithm mimics the foraging behavior of honey bees while searching food for them. ABC is a simple and population based optimization algorithm. Here the population consists of possible solutions in terms of food sources for honey bees whose fitness is regulated in terms of nectar amount which the food source contains. The swarm updating in ABC is due to two processes namely, the variation process and the selection process which are responsible for exploration and exploitation, respectively. However the ABC achieves a good solution but, like the other optimization algorithms, it has also problem of premature convergence and stagnation. On the other part, it is also required to tune the ABC control parameters based on problem. Also literature says that basic ABC itself has some drawbacks like stop proceeding toward the global optimum even though the population has not converged to a local optimum (Karaboga and Akay 2009) and it is observed that the position update equation of ABC algorithm is inefficient to balance exploration and exploitation (Zhu and Kwong 2010). Therefore these drawbacks require a modification in position update equation of ABC in order to make it capable to balance exploration and exploitation. These drawbacks have also addressed in earlier research. To enhance the exploitation, Wei-feng Gao and Liu (2011) improved position update equation of ABC such that the bee searches only in neighborhood of the previous iteration's best solution. Anan Banharnsakun et al. (2011) proposed the best-so-far selection in ABC algorithm and incorporated three major changes: The best-so-far method, an adjustable search radius, and an objective-value-based comparison in ABC. To solve constrained optimization problems, D. Karaboga and B. Akay (2011) used Deb's rules consisting of three simple heuristic rules and a probabilistic selection scheme in ABC algorithm. Dervis Karaboga (2005) examined and suggested that the limit should be taken as $S N \times D$, where, $S N$ is the population size and $D$ is the dimension of the problem and coefficient $\phi_{i j}$ in position update equation should be adopted in the
range of $[-1,1]$. Further, Kang et al. (2011) introduced exploitation phase in ABC using Rosenbrock's rotational direction method and named modified ABC as Rosenbrock ABC (RABC). Qingxian and Haijun proposed a new initialization scheme by making the initial group symmetrical and to increase the ABC's convergence, they applied Boltzmann selection mechanism in place of roulette wheel selection (Haijun and Qingxian 2008). Tsai et al. (2009) proposed Interactive $A B C$ (IABC) in which author applied the roulette wheel selection to select onlooker bees and he also used Newtonian law of universal gravitation to enhance the exploitation in onlooker bee phase. Zhu et al. (2010) also modified the position update equation of basic ABC by incorporated best individual member information in its position update equation. Baykasoglu et al. (2007) solved the generalized assignment problem by hybriding the ABC algorithm with shift neighborhood searches and greedy randomized adaptive search heuristic. Furthermore, Bahriye Akay and Dervis Karaboga (2010) also presented an improved ABC to solve real-parameter optimization problems and they also analyzed the effects of the perturbation rate, the scaling factor limit parameter on realparameter optimization. Some other important improved versions of ABC can be found in Bansal et al. (2013); Sharma et al. (2013); Bansal et al. (2013); Jadon et al. (2014).

Inspired from the article "Differential evolution using a neighborhood-based mutation operator (DEGL) "(Das et al. 2009), we propose a new position update equation for employed bees in ABC which linearly incorporate two components namely local and global components in its swarm updating process in order to set a trade off between the exploration and exploitation capabilities of ABC. Here, local component is responsible for explorative moves and has better chance of locating the minima of test function. On the other hand, global component is responsible for exploitive moves and rapidly converges to a minimum of the test function.

Rest of the paper is organized as follows. Next Sect. 2 explains standard $A B C$ algorithm. In Sect. 3, proposed modified ABC (ABCGLN) is described. Section 4 details about the benchmark mathematical optimization functions used in this article and also statistical analysis to compare the performance of the proposed strategy with respect to other recent ABC variants. Finally, in Sect. 5, paper is concluded.

## 2 Artificial Bee Colony(ABC) algorithm

The ABC algorithm is a population based recent swarm intelligence based algorithm which is inspired by food
foraging behavior of honey bees. In ABC , each solution is known as food source of honey bees whose fitness is determined in terms of the quality of the food source. Artificial Bee Colony is made of three group of bees: employed bees, onlooker bees and scout bees. The number of employed and onlooker bees is equal. The employed bees searches the food source in the environment and store the information like the quality and the distance of the food source from the hive. Onlooker bees wait in the hive for employed bees and after collecting information from them, they start searching in neighborhood of that food sources which are having better nectar. If any food source is abandoned then scout bee finds new food source randomly in search space. While searching the solution of any optimization problem, ABC algorithm first initializes ABC parameters and swarm then it requires the repetitive iterations of the three phases namely employed bee phase, onlooker bee phase and scout bee phase. In ABC, first initialization of the solutions is done as:

### 2.1 Initialization of the swarm

If $D$ is the number of variables in the optimization problem then each food source $x_{i}(i=1,2, \ldots, S N)$ is a $D$-dimensional vector among the $S N$ food sources and is generated using a uniform distribution as:
$x_{i j}=x_{\text {minj }}+\operatorname{rand}[0,1]\left(x_{\text {maxj }}-x_{\text {minj }}\right)$
here $x_{i}$ represents the $i t h$ food source in the swarm, $x_{\text {minj }}$ and $x_{\operatorname{maxj}}$ are bounds of $x_{i}$ in $j$ th dimension and rand $[0,1]$ is a uniformly distributed random number in the range $[0,1]$. After initialization phase $A B C$ requires the cycle of the three phases namely employed bee phase, onlooker bee phase and scout bee phase to be executed.

### 2.2 Employed bees phase

In this phase, ith candidate's position is updated using following equation:
$v_{i j}=x_{i j}+\phi_{i j}\left(x_{i j}-x_{k j}\right)$
here $k \in\{1,2, \ldots, S N\}$ and $j \in\{1,2, \ldots, D\}$ are randomly chosen indices and $k \neq i . \phi_{i j}$ is a random number in the
range $[-1,1]$. After generating new position, the position with better fitness between the newly generated and old one is selected.

### 2.3 Onlooker bees phase

In this phase, employed bees share the information associated with their food sources like quality (nectar) and position of the food source with the onlooker bees in the hive. Onlooker bees evaluate the available information about the food source and based on its fitness they select solutions with probability prob $_{i}$. Here prob $_{i}$ can be calculated as function of fitness (there may be some other):
$\operatorname{prob}_{i}(G)=\frac{0.9 \times \text { fitness }_{i}}{\text { maxfit }}+0.1$,
here fitness ${ }_{i}$ is the fitness value of the $i$ th solution and maxfit is the maximum fitness amongst all the solutions. Based on this probability, onlooker selects a solution and modifies it using the same Eq. (2) as in employed bee phase. Again by applying greedy selection, if the fitness is higher than the previous one, the onlooker bee stores the new position in its memory and forgets the old one.

### 2.4 Scout bees phase

If for a predetermined number of cycles, any bee's position is not getting updated then that food source is taken to be abandoned and this bee becomes scout bee. In this phase, the abandoned food source is replaced by a randomly chosen food source within the search space. In ABC , the number of cycles after which a particular food source becomes abandoned is known as limit and is a crucial control parameter. In this phase the abandoned food source $x_{i}$ is replaced by a randomly chosen food source within the search space using the Eq. (1) as in initialization phase.

### 2.5 Main steps of the ABC algorithm

The pseudo-code of the ABC is shown in Algorithm 1 (Karaboga and Akay 2009).

```
Algorithm 1 Artificial Bee Colony Algorithm:
    Initialize the parameters;
    while Termination criteria is not satisfied do
        Step 1: Employed bee phase for generating new food sources;
        Step 2: Onlooker bees phase for updating the food sources depending on their nectar amounts;
        Step 3: Scout bee phase for discovering the new food sources in place of abandoned food sources;
        Step 4: Memorize the best food source found so far;
    end while
    Output the best solution found so far.
```


## 3 Artificial bee colony algorithm with global and local neighborhoods

For an efficient optimization algorithm, two concepts exploration and exploitation should be well balanced which are actually contradictory to each other in nature. Exploration means the ability of exploring diverse regions of the search space and exploitation means refining already explored regions. In ABC, at any instance, a solution is updated through information flow from other solutions of the swarm. This position updating process uses a linear combination of current position of the potential solution which is going to be updated and position of a randomly selected solution as step size with a random coefficient $\phi_{i j} \in[-1,1]$. This process plays an important role to decide the quality of new solution. If the current solution is far from randomly selected solution and absolute value of $\phi_{i j}$ is also high then the change will be large enough to jump the true optima. On the other hand, small change will decrease the convergence rate of whole ABC process. Further, It is also suggested in literature (Karaboga and Akay 2009; Zhu and Kwong 2010) that basic ABC itself has some drawbacks, like stop proceeding toward the global optimum even though the population has not converged to a local optimum. Karaboga and Akay (2009) also analyzed the various variants of ABC and found that the ABC in its current form shows poor performance and remains inefficient to balance the exploration and exploitation capabilities of the search space. Consequently, convergence speed of ABC is also deteriorated.

In above context, we are proposing a modified ABC by incorporating local and global neighborhoods concept in its position update equation of employed bee phase. This modified ABC is now onward called as ABCGLN. The proposed modified position update equation is the inspiration from DEGL of Das et al. (2009). In DEGL, author proposed a very strong and promising DE version which includes two neighborhood models; the local neighborhood model and the global neighborhood model. In local neighborhood model each member is updated through best solution found so far in its sub population (i.e, local neighborhood), while in the global neighborhood model each member takes the advantage of best solution found so far in the whole population. We adopted both neighborhood strategies of DEGL in the proposed ABCGLN algorithm.

In ABCGLN, the neighborhood and neighborhood structure of any bee are crucial parts which need to be focussed. It should be clear here that the neighborhood structure of any solution is static and has been defined on the set of indices of the solutions. Neighborhood of a solution is the set of other solutions to which it is connected. For global neighborhood model, neighborhood structure is the star topology i.e, each solution is connected to each other
solution and therefore cardinality of neighborhood of any solution in this model is equal to the number of food sources. On the other hand, for local neighborhood model, ring topology is adopted as neighborhood structure and cardinality of neighborhood is considered as $10 \%$ of the number of food sources i.e, neighborhood of each solution includes $10 \%$ solutions based on their indices as $5 \%$ from the forward side and $5 \%$ from the backward side. Here, it should be noticed that solutions belonging to a local neighborhood are not necessarily local in the sense of their geographical nearness or similar fitness values but, the overlapping neighborhoods based on the indices of the swarm members have been considered in ABC as shown in Fig. 1. For example, if population size is 60 then local neighborhood of 20 th indexed solution will include solutions from indexed 17 th to 23 th, similarly 21 st indexed solution's neighborhood includes solutions from indexed 18th to 24 th (see Fig. 1). In ABCGLN, for each employed bee, two components a local and a global component are created. The local component for a bee is created based on best solution position in that bee's local neighborhood and two other solutions selected randomly from this neighborhood while global component is created based on best solution of whole swarm and two other solutions selected randomly from the swarm i.e, from the global neighborhood. The mathematical equations for these components are as follows:

$$
\begin{align*}
L_{i j} & =x_{i j}+\left(\text { prob }_{i}\right)\left(x_{l j}-x_{i j}\right)+\phi_{i j}\left(x_{r 1 j}-x_{r 2 j}\right) \\
G_{i j} & =x_{i j}+\left(1-\operatorname{prob}_{i}\right)\left(x_{g j}-x_{i j}\right)+\phi_{i j}\left(x_{R 1 j}-x_{R 2 j}\right) \tag{4}
\end{align*}
$$

where, each $j$ represents the $j$ th dimension of the each position, $L_{i}$ and $G_{i}$ are the local and global components respectively. $x_{l}$ and $x_{g}$ are respectively the best positions in local and global neighborhoods of $i t h$ solution. $x_{r 1}, x_{r 2}$ are the two neighbors chosen randomly from the local neighborhood of $i$ th solution such that $r 1 \neq r 2 \neq i . x_{R 1}, x_{R 2}$ are the two neighbors chosen randomly from the global neighborhood i.e, from whole swarm such that $R 1 \neq R 2 \neq i . \phi_{i j}$ is random number in $[-1,1]$ and $\operatorname{prob}_{i}$ is the probability of ith solution as in Eq. (3). Here in both equations the third term i.e, difference between two randomly selected neighbors is added so that individual member should not follow only the best position in order to prevent the swarm from getting trapped in local minima. Also second term in both the equations is multiplied by their probabilities which is a function of fitness. It is clear that solution with better fitness will give more weightage to its local best than the global best in order to prevent the swarm to converge too quickly as global best may be a local optima in initial stages. On the other hand the solution with low fitness will give priority to global best more than the local best as there surroundings are not good enough so they follow better directions. Finally
both the local and the global components together guide the new direction for employed bees as follows: for ith employed bee, the new position is,
$v_{i j}=\left(p r o b_{i}\right) \times G_{i j}+\left(1-\right.$ prob $\left._{i}\right) \times L_{i j}$

Now based on greedy selection, employed bee selects one between the old and new positions. In this way, the

The ABCGLN is composed of three phases: employed bee phase, onlooker bee phase and scout bee phase. Only the employed bee phase is changed and other phases; onlooker and scout bee remain same as in basic ABC. Here the new position search strategy (explained in Eqs. (4) and (5)) is proposed for employed bees. The pseudo-code of the proposed ABCGLN algorithm is shown in Algorithm 3.

```
Algorithm 3 ABC with Global and Local neighborhoods (ABCGLN):
    Initialize the population and control parameters;
    while Termination criteria is not satisfied do
        Memorize the best food source found so far;
        Calculate prob \({ }_{i}\) using Equation (3) for each bee;
        Employed bee phase: apply Algorithm (2) to generate new food sources for each employed bee.
        Onlooker bees phase: use Equation (2) to update the food sources for particular bees selected based on prob \(_{i}\);
        Scout bee phase: use Equation (1) to determine the new food sources for exhausted food sources.
    end while
    Return the best solution found so far.
```

involvement of weighted linear sum of both local and global best components, is expected to balance the exploration and exploitation capabilities of ABC algorithm as the local best components of respective bees explore the search space or tries to identify the most promising search space regions, while the global best component will exploit the identified search space. The pseudo code of implementation of this whole neighborhood concept in employed bee phase of ABC is explained in Algorithm 2. Algorithm 2 introduced a new parameter $c r$. For each dimension of employed bee, a random number between 0 and 1 is generated and if it is less than cr value then that particular dimension will be changed. Actually, in basic ABC, only one random dimension of employed bee is changed for employed in order to generate neighborhood solution but in proposed algorithm ABCGLN, the number of dimensions to be changed are selected based on probability $c r$. In ABCGLN, $c r$ is set to a low value 0.3 based on its sensitive analysis carried out in Fig. 2.

## 4 Experimental results and discussion

### 4.1 Test problems under consideration

To validate the effectiveness of the proposed algorithm ABCGLN, 24 mathematical optimization problems ( $f_{1}$ to $f_{24}$ ) of different characteristics and complexities are taken into consideration (listed in Table 1). These all problems are continuous in nature. Test problems $f_{1}-f_{17}$ and $f_{22}-f_{24}$ are taken from Ali et al. (2005) and test problems $f_{18}-f_{21}$ are taken from Suganthan et al. (2005) with the associated offset values. Table 1 includes various kind of unimodal, multimodal, separable and non separable test problems. A unimodal function $f(x)$ has a single extremum (minimum or maximum in the range specified for $x$ ) while, a function having more than one peaks in the search space i.e., local extremum, is said multimodal. Algorithms are tested on this type of functions to check their ability to coming out of local minima.

```
Algorithm 2 Proposed employed bee phase for ABCGLN:
    Input employed bee \(i, p r o b_{i}\) and best solution \(g\) found so far in whole swarm;
    Find best solution \(l\) in local neighborhood of \(i^{t h}\) bee;
    Select two neighbors \(x_{r 1}, x_{r 2}\) randomly from the local neighborhood of \(i^{\text {th }}\) bee such that \(r 1 \neq r 2 \neq i\);
    Select two neighbors \(x_{R 1}, x_{R 2}\) randomly from whole swarm such that \(R 1 \neq R 2 \neq i\);
    for (each dimension \(j\) ) do
        if (rand \(<c r\) ) then \(\{\#\) rand is a uniform random number between 0 and 1\(\}\)
            Generate local component \(L_{i j}\) and global component \(G_{i j}\) using Equation (4);
            Generate new position \(v_{i j}\) using Equation (5);
        else
            \(v_{i j}=x_{i j} ;\)
        end if
    end for
    Return better of old position \(x_{i j}\) and new position \(v_{i j}\) for employed bee \(i\);
```


### 4.2 Experimental setting

The results obtained from the proposed ABCGLN are stored in the form of success rate, average number of function evaluations, standard deviation of the fitness and mean error. Results for test problems (Table 1) are also obtained from the basic ABC and recent variants of ABC named Gbest-guided ABC (GABC) (Zhu and Kwong 2010), Best-So-Far ABC (BSFABC) (Banharnsakun et al. 2011) and Modified ABC (MABC) (Akay and Karaboga 2010) for the comparison purpose. The following parameter setting is adopted while implementing our proposed and other considered algorithms to solve the problems:

- The number of simulations/run $=100$,
- Colony size $N P=50$ (Diwold et al. 2011; El-Abd 2011) and Number of food sources $S N=N P / 2$,
- $\phi_{i j}=\operatorname{rand}[-1,1]$ and limit $=$ dimension $\times$ number of food sources $=D \times S N$ (Karaboga and Akay 2011; Akay and Karaboga 2010),
- The terminating criteria: Either acceptable error (mentioned in Table 1) meets or maximum number of function evaluations (which is set to be 200000) is reached,


Fig. 1 Local neighborhood topology for radius 3
times, algorithm achieved the function optima with acceptable error in 100 runs, $A F E$ is the average number of function evaluations of 100 runs by the algorithm to reach at the termination criteria and $M E$ is the mean of absolute difference between exact and obtained solutions over 100 runs. Mathematically $M E$ and $A F E$ are defined as:

$$
\begin{aligned}
M E & =\frac{\sum_{i=1}^{100} \mid \text { Exact solution }- \text { Obtained solution } \mid \text { for run } \mathrm{i}}{100} \\
A F E & =\frac{\sum_{i=1}^{100} \text { Number of function evaluations to meet the termination criteria for run } \mathrm{i}}{100}
\end{aligned}
$$

- The parameter $c r$ in Algorithm 2 is set to 0.3 based on its sensitive analysis in range $[0.1,1]$ as explained in the Fig. 2. Figure 2 shows a graph between cr and the sum of the successful runs for all the considered problems. It is clear that $c_{r}=0.3$ provides the highest success rate.
- Parameter settings for the other considered algorithms $\mathrm{ABC}, \mathrm{GABC}, \mathrm{BSFABC}$ and MABC are adopted from their original articles.


### 4.3 Results analysis of experiments

Tables 2, 3 and 4 present the numerical results comparison of all considered algorithms for benchmark problems $f_{1}-$ $f_{24}$ with the experimental settings shown in Sect. 4.2. Tables 2, 3 and 4 show the results of the proposed and other considered algorithms in terms of success rate (SR), average number of function evaluations (AFE) and mean error (ME) respectively. Here $S R$ represents the number of

It can be seen from Table 2 that ABCGLN is more reliable as it has achieved higher or equal success rate than all the considered algorithms on all functions except $f_{3}$. It can be observed from Table 3 that most of the time, ABCGLN is more efficient as it took less number of function evaluations on 23 out of 24 test problems than the ABC and other considered modified ABC algorithms. Table 4 shows a clear superiority of ABCGLN in terms of accuracy as ABCGLN's mean error is less than the mean error achieved by the other algorithms on 19 out of 24 test functions. While comparing all three factors simultaneously then it can be analyzed that ABCGLN costs less than the considered algorithms over 19 test problems $\left(f_{1}-f_{2}, f_{4}-f_{7}, f_{9}-f_{10}, f_{12}-f_{13}, f_{15}-f_{20}\right.$ and $\left.f_{22}-f_{24}\right)$ out of 24 test functions. As these functions include unimodel, multimodel, separable, non separable, lower and higher dimensional functions, it can be stated that AB CGLN balances the exploration and exploitation capabilities efficiently on most of the functions. This should be
Table 1 Benchmark functions used in experiments

| Test problem | Objective function | Search range | Optimum value | D | C | AE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sphere | $f_{1}(x)=\sum_{i=1}^{D} x_{i}^{2}$ | [-5.12 5.12] | $f(\boldsymbol{O})=0$ | 30 | US | $1.0 E-05$ |
| Griewank | $f_{2}(x)=1+\frac{1}{4000} \sum_{i=1}^{D} x_{i}^{2}-\prod_{i=1}^{D} \cos \left(\frac{x_{1}}{\sqrt{i}}\right)$ | [-600 600] | $f(\boldsymbol{O})=0$ | 30 | MN | $1.0 E-05$ |
| Rosenbrock | $f_{3}(x)=\sum_{i=1}^{D}\left(100\left(x_{i+1}-x_{i}^{2}\right)^{2}+\left(x_{i}-1\right)^{2}\right)$ | [-30 30] | $f(\boldsymbol{I})=0$ | 30 | UN | $1.0 E-02$ |
| Ackley | $\begin{aligned} & f_{4}(x)=-20+e+\exp \left(-\frac{0.2}{D} \sqrt{\sum_{i=1}^{D} x_{i}^{3}}\right) \\ & -\exp \left(\frac{1}{D} \sum_{i=1}^{D} \cos \left(2 \pi x_{i}\right) x_{i}\right) \end{aligned}$ | $\left[\begin{array}{lll}-1 & 1\end{array}\right]$ | $f(\boldsymbol{O})=0$ | 30 | MN | $1.0 E-05$ |
| Cosine Mixture | $f_{5}(x)=\sum_{i=1}^{D} x_{i}{ }^{2}-0.1\left(\sum_{i=1}^{D} \cos 5 \pi x_{i}\right)+0.1 D$ | $\left[\begin{array}{lll}-1 & 1\end{array}\right]$ | $f(\boldsymbol{O})=-D \times 0.1$ | 30 | MS | $1.0 E-05$ |
| Exponential | $f_{6}(x)=-\left(\exp \left(-0.5 \sum_{i=1}^{D} x_{i}{ }^{2}\right)\right)+1$ | $\left[\begin{array}{lll}-1 & 1\end{array}\right]$ | $f(\boldsymbol{O})=-1$ | 30 | MN | $1.0 E-05$ |
| Zakharov | $f_{7}(x)=\sum_{i=1}^{D} x_{i}{ }^{2}+\left(\sum_{i=1}^{D} \frac{i x_{2}}{2}\right)^{2}+\left(\sum_{i=1}^{D} \frac{i x_{1}}{2}\right)^{4}$ | [-5.12 5.12] | $f(\boldsymbol{O})=0$ | 30 | MN | $1.0 E-02$ |
| Brown3 | $\left.f_{8}(x)=\sum_{i=1}^{D-1}\left(x_{i}{ }^{2(x+1)}\right)^{2}+1+x_{i+1}{ }^{2 x_{i}{ }^{2}+1}\right)$ | [-14] | $f(\boldsymbol{O})=0$ | 30 | UN | $1.0 E-05$ |
| Schewel | $f_{9}(x)=\sum_{i=1}^{D}\left\|x_{i}\right\|+\prod_{i=1}^{D}\left\|x_{i}\right\|$ | [-10 10] | $f(\boldsymbol{O})=0$ | 30 | UN | $1.0 E-05$ |
| Salomon Problem | $f_{10}(x)=1-\cos \left(2 \pi \sqrt{\sum_{i=1}^{D} x_{i}^{2}}\right)+0.1\left(\sqrt{\sum_{i=1}^{D} x_{i}^{2}}\right)$ | [-100 100] | $f(\boldsymbol{O})=0$ | 30 | MN | $1.0 E-01$ |
| Powel Sum of different powers | $f_{11}(x)=\sum_{i=1}^{D}\left\|x_{i}\right\|^{i+1}$ | [-11] | $f(\boldsymbol{O})=0$ | 30 | US | $1.0 E-05$ |
| Inverted cosine wave | $\begin{gathered} f_{12}(x)=-\sum_{i=1}^{D-1}\left(\exp \left(\frac{\left(x_{i}^{2}+x_{i+1}^{2}+0.5 x_{i} x_{i+1}\right.}{8}\right) \times \mathrm{I}\right) \\ \text { where, } \mathrm{I}=\cos \left(4 \sqrt{x_{i}^{2}+x_{i+1}^{2}+0.5 x_{i} x_{i+1}}\right) \end{gathered}$ | [-5 5] | $f(\boldsymbol{O})=-\mathrm{D}+1$ | 10 | MN | $1.0 E-05$ |
| Neumaier 3 Problem (NF3) | $f_{13}(x)=\sum_{i=1}^{D}\left(x_{i}-1\right)^{2}-\sum_{i=2}^{D} x_{i} x_{i-1}$ | $\left[-D^{2} D^{2}\right]$ | $f_{\text {min }}=-\frac{(D(D+4)(D-1))}{6}$ | 10 | UN | $1.0 E-01$ |
| Levy montalvo 2 | $f_{14}(x)=0.1\left(\sin ^{2}\left(3 \pi x_{1}\right)+\sum_{i=1}^{D-1}\left(x_{i}-1\right)^{2} \times\left(1+\sin ^{2}\left(3 \pi x_{i+1}\right)\right)+\left(x_{D}-1\right)^{2}\left(1+\sin ^{2}\left(2 \pi x_{D}\right)\right)\right.$ | [-5 5] | $f(\boldsymbol{I})=0$ | 30 | MN | $1.0 E-05$ |
| Beale | $f_{15}(x)=\left[1.5-x_{1}\left(1-x_{2}\right)\right]^{2}+\left[2.25-x_{1}\left(1-x_{2}^{2}\right)\right]^{2}+\left[2.625-x_{1}\left(1-x_{2}^{3}\right)\right]^{2}$ | [-4.5 4.5] | $f(3,0.5)=0$ | 2 | UN | $1.0 E-05$ |
| Colville | $f_{16}(x)=100\left[x_{2}-x_{1}^{2}\right]^{2}+\left(1-x_{1}\right)^{2}+90\left(x_{4}-x_{3}^{2}\right)^{2}+\left(1-x_{3}\right)^{2}+10.1\left[\left(x_{2}-1\right)^{2}+\left(x_{4}-1\right)^{2}\right]+19.8\left(x_{2}-1\right)\left(x_{4}-1\right)$ | [-10 10] | $f(\boldsymbol{I})=0$ | 4 | MN | $1.0 E-05$ |
| Kowalik | $f_{17}(x)=\sum_{i=1}^{11}\left[a_{i}-\frac{x_{1}\left(b_{2}^{2}+b_{1}+x_{2}\right)}{b_{i}^{2}+b_{x}, x_{3}+x_{1}}\right]^{2}$ | [-5 5] | $\begin{gathered} f(0.192833,0.190836,0.123117, \\ 0.135766)=0.000307486 \end{gathered}$ | 4 | MN | $1.0 E-05$ |
| Shifted Rosenbrock | $f_{18}(x)=\sum_{i=1}^{D-1}\left(100\left(z_{i}^{2}-z_{i+1}\right)^{2}+\left(z_{i}-1\right)^{2}\right)+f_{\text {biass }}, z=x-o+1, x=\left[x_{1}, x_{2}, \ldots, x_{D}\right], o=\left[o_{1}, o_{2}, \ldots o_{D}\right]$ | [-100 100] | $f(o)=f_{\text {bias }}=390$ | 10 | MN | $1.0 E-01$ |
| Shifted Sphere | $f_{19}(x)=\sum_{i=1}^{D} z_{i}^{2}+f_{\text {bias }}, z=x-o, x=\left[x_{1}, x_{2}, \ldots, x_{D}\right], o=\left[o_{1}, o_{2}, \ldots o_{D}\right]$ | [-100 100] | $f(o)=f_{\text {bias }}=-450$ | 10 | US | $1.0 E-05$ |
| Shifted Griewank | $f_{20}(x)=\sum_{i=1}^{D} \frac{z_{i}^{2}}{4000}-\prod_{i=1}^{D} \cos \left(\frac{z_{i}}{\sqrt{i}}\right)+1+f_{\text {biass }}, z=(x-o), x=\left[x_{1}, x_{2}, \ldots x_{D}\right], o=\left[o_{1}, o_{2}, \ldots o_{D}\right]$ | [-600 600] | $f(o)=f_{\text {bias }}=-180$ | 10 | MN | $1.0 E-05$ |
| Shifted Ackley | $\begin{aligned} f_{21}(x) & =-20 \exp \left(-0.2 \sqrt{\frac{1}{D} \sum_{i=1}^{D} z_{i}^{2}}\right)-\exp \left(\frac{1}{D} \sum_{i=1}^{D} \cos \left(2 \pi z_{i}\right)\right)+20+e+f_{\text {bias }}, \\ z & =(x-o), x=\left(x_{1}, x_{2}, \ldots \ldots x_{D}\right), o=\left(o_{1}, o_{2}, \ldots \ldots . o_{D}\right) \end{aligned}$ | [-32 32] | $f(o)=f_{\text {bias }}=-140$ | 10 | MN | $1.0 E-05$ |
| Goldstein-Price | $\begin{aligned} f_{22}(x)= & \left(1+\left(x_{1}+x_{2}+1\right)^{2} \cdot\left(19-14 x_{1}+3 x_{1}^{2}-14 x_{2}\right.\right. \\ & \left.\left.+6 x_{1} x_{2}+3 x_{2}^{2}\right)\right) \cdot\left(30+\left(2 x_{1}-3 x_{2}\right)^{2} \cdot\left(18-32 x_{1}+12 x_{1}^{2}+48 x_{2}-36 x_{1} x_{2}+27 x_{2}^{2}\right)\right) \end{aligned}$ | [-2 2] | $f(0,-1)=3$ | 2 | MN | $1.0 E-14$ |
| Meyer and Roth |  | [-1010] | $f(3.13,15.16,0.78)=0.4 \times 10^{-4}$ | 3 | UN | 1.0E-03 |
| Moved axis parallel hyper-ellipsoid | $f_{24}(x)=\sum_{i=1}^{D} 5 i \times x_{i}^{2}$ | [-5.12 5.12] | $f(x)=0 ; x(i)=5 \times i, i=1: D$ | 30 | MS | $1.0 E-15$ |

[^1]

Fig. 2 Effect of parameter $c r$
noticed that ABCGLN performed worst than all considered algorithms on unimodel nonseparable test function $f_{3}$. Some other statistical tests like the Mann-Whitney U rank sum test, acceleration rate (AR) (Rahnamayan et al. 2008), boxplots and performance indices (Bansal and Sharma 2012) have also been done in next subsection to analyze the algorithms output more intensively.

### 4.4 Statistical analysis

Since the empirical distribution of results can efficiently be represented by boxplot (Williamson et al. 1989), the boxplots for success rate (SR), average number of function evaluations (AFE) and mean error (ME) for ABCGLN and considered algorithms have been represented in Fig. 3 (a), (b), (c). Figures 3 (a), (b), (c) show that ABCGLN is cost effective in terms of SR, AFE and ME as the interquartile range and median are very low for ABCGLN. It can be observed from Fig. 3(a) that box of ABCGLN for SR is not being seen, as in almost all cases, ABCGLN achieved the $100 \%$ success rate so there is negligible variation which boxplot is unable to represent and hence is showing a constant line segment instead of variation box. Boxplots for average number of function evaluations in Fig. 3(b) shows clear superiority of ABCGLN over other considered algorithms as the interquartile range and median for ABCGLN are very low. Boxplots for mean error in Fig. 3(c) represents that ABCGLN and MABC achieved almost equal and less average error than the other algorithms.

Now, it is clear from boxplot for AFE in Figure 3(b) that results of ABCGLN differs from the other algorithms. But to check, significant difference between ABCGLN's and other algorithm's output or the difference is due to some randomness, we applied another statistical test namely, the Mann-Whitney U rank sum (Mann and Whitney 1947) on AFE. We have chosen this non parametric test to compare the performance of the algorithms as boxplots of Fig. 3 (b)
show that average number of function evaluations used by the considered algorithms to solve the different problems are not normally distributed. The test is performed at $5 \%$ level of significance $(\alpha=0.05)$ between our proposed ABCGLN and each other considered algorithm.

The results of the Mann-Whitney U rank sum test for the average function evaluations of 100 simulations is presented in Table 5. First we checked that two data sets are significantly different or not through Mann-Whitney U rank sum test. If significant difference is not seen (i.e., the null hypothesis is accepted) then sign ' $=$ ' appears and when significant difference is observed i.e., the null hypothesis is rejected then we compared the average number of function evaluations. And we use signs ' + ' and ' - ' for the case where ABCGLN takes less or more average number of function evaluations than the other algorithms, respectively. Therefore in Table 5, ' + ' indicates that ABCGLN is significantly better and '-' shows that ABCGLN is significantly worse, while ' $=$ ' means that performance of both algorithms is equivalent. Since Table 5 contains 91 ' + ' signs out of 96 comparisons, it can be concluded that ABCGLN performance is significantly cost effective than other considered algorithms over test problems of Table 1.

Further, to compare the considered algorithms by giving weighted importance to $\mathrm{SR}, \mathrm{AFE}$ and ME, performance indices (PIs) are calculated (Bansal and Sharma 2012). The values of PI for the ABCGLN and other considered algorithms, are calculated using following equations:
$P I=\frac{1}{N_{p}} \sum_{i=1}^{N_{p}}\left(k_{1} \alpha_{1}^{i}+k_{2} \alpha_{2}^{i}+k_{3} \alpha_{3}^{i}\right)$
$\quad \begin{gathered}\text { where } \alpha_{1}^{i}=\frac{S r^{i}}{T r^{i}} ; \alpha_{2}^{i}=\left\{\begin{array}{cl}\frac{M f^{i}}{A f^{i}}, & \text { if } S r^{i}>0 . \\ 0, & \text { if } S r^{i}=0 .\end{array} ; \text { and } \alpha_{3}^{i}=\frac{M o^{i}}{A o^{i}} \text {, } 2, \ldots, N_{p}\right.\end{gathered}$

- $k_{1}, k_{2}$ and $k_{3}$ are the weights assigned to SR, AFE and ME respectively, where $k_{1}+k_{2}+k_{3}=1$ and $0 \leq k_{1}, k_{2}, k_{3} \leq 1$.
- $S r^{i}=$ Successful simulations/runs of $i t h$ problem.
- $\operatorname{Tr}^{i}=$ Total simulations of $i t h$ problem.
- $M f^{i}=$ Minimum of mean number of function evaluations performed to achieve the required solution of $i t h$ problem.
- $A f^{i}=$ Average number of function evaluations performed to achieve the required solution of $i t h$ problem.
- $M o^{i}=$ Minimum of mean error obtained for the ith problem.
- $A o^{i}=$ Mean Error achieved by an algorithm for the ith problem.
- $N_{p}=$ Total number of optimization problems evaluated.
To calculate the PIs, equal weights are assigned to two out of SR, AFE and ME variables and weight of the remaining

Table 2 Success Rate (SR) achieved by the algorithms in 100 runs, TP: Test Problems

| TP | ABC | BSFABC | GABC | MABC | ABCGLN | TP | ABC | BSFABC | GABC | MABC | ABCGLN |
| :--- | ---: | ---: | ---: | ---: | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $f_{1}$ | 100 | 100 | 100 | 100 | 100 | $f_{13}$ | 3 | 9 | 23 | 30 | 89 |
| $f_{2}$ | 97 | 100 | 100 | 100 | 100 | $f_{14}$ | 100 | 100 | 100 | 100 | 100 |
| $f_{3}$ | 22 | 24 | 25 | 10 | 9 | $f_{15}$ | 96 | 98 | 100 | 100 | 100 |
| $f_{4}$ | 100 | 100 | 100 | 100 | 100 | $f_{16}$ | 1 | 31 | 33 | 39 | 93 |
| $f_{5}$ | 100 | 100 | 100 | 100 | 100 | $f_{17}$ | 21 | 57 | 88 | 27 | 94 |
| $f_{6}$ | 100 | 100 | 100 | 100 | 100 | $f_{18}$ | 21 | 24 | 56 | 34 | 100 |
| $f_{7}$ | 0 | 0 | 0 | 0 | 100 | $f_{19}$ | 100 | 100 | 100 | 100 | 100 |
| $f_{8}$ | 100 | 100 | 100 | 100 | 100 | $f_{20}$ | 83 | 82 | 98 | 92 | 98 |
| $f_{9}$ | 100 | 100 | 100 | 100 | 100 | $f_{21}$ | 100 | 100 | 100 | 100 | 100 |
| $f_{10}$ | 57 | 63 | 97 | 99 | 100 | $f_{22}$ | 53 | 57 | 53 | 54 | 100 |
| $f_{11}$ | 100 | 100 | 100 | 100 | 100 | $f_{23}$ | 100 | 99 | 100 | 100 | 100 |
| $f_{12}$ | 84 | 84 | 98 | 100 | 100 | $f_{24}$ | 99 | 100 | 100 | 100 | 100 |

Table 3 Average number of function evolutions (AFE) done by the algorithms in 100 runs, TP: Test Problems

| TP | ABC | BSFABC | GABC | MABC | ABCGLN | TP | ABC | BSFABC | GABC | MABC | ABCGLN |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $f_{1}$ | 20639 | 29864 | 24937 | 23004 | $\mathbf{9 5 9 1}$ | $f_{13}$ | 198760 | 194001 | 186012 | 169918 | $\mathbf{1 4 1 6 2 6}$ |
| $f_{2}$ | 46004 | 43115 | 33900 | 42835 | $\mathbf{2 4 2 1 0}$ | $f_{14}$ | 28861 | 21968 | 26702 | 20921 | $\mathbf{1 0 0 1 2}$ |
| $f_{3}$ | 183117 | 180542 | $\mathbf{1 7 5 9 1 1}$ | 189716 | 186909 | $f_{15}$ | 28215 | 27917 | 14007 | 10818 |  |
| $f_{4}$ | 49107 | 42879 | 47858 | 43630 | $\mathbf{1 8 0 4 6}$ | $f_{16}$ | 199667 | 159666 | 160337 | 140006 | $\mathbf{6 3 4 7}$ |
| $f_{5}$ | 23016 | 31789 | 28572 | 22764 | $\mathbf{9 9 6 2}$ | $f_{17}$ | 179860 | 142527 | 99409 | 170962 | $\mathbf{5 9 3 9}$ |
| $f_{6}$ | 16836 | 16791 | 22576 | 16305 | $\mathbf{7 3 1 9}$ | $f_{18}$ | 181580 | 171251 | 110077 | 150959 | $\mathbf{8 0 5 2 3}$ |
| $f_{7}$ | 200000 | 200000 | 200000 | 200000 | $\mathbf{1 3 1 2 0 7}$ | $f_{19}$ | 9247 | 12203 | 8839 | 8664 | $\mathbf{6 0 7 3}$ |
| $f_{8}$ | 21271 | 31445 | 25837 | 22992 | $\mathbf{1 0 3 2 4}$ | $f_{20}$ | 88929 | 99271 | 38443 | 81323 | $\mathbf{2 7 6 5 3}$ |
| $f_{9}$ | 52841 | 41807 | 52705 | 32889 | $\mathbf{1 8 7 2 6}$ | $f_{21}$ | 17662 | 31160 | 16069 | 14217 | $\mathbf{7 9 6 1}$ |
| $f_{10}$ | 190143 | 157648 | 122536 | 26303 | $\mathbf{1 1 3 2 8}$ | $f_{22}$ | 126814 | 94450 | 96819 | 98001 | $\mathbf{8 6 9 7}$ |
| $f_{11}$ | 16096 | 14647 | 15017 | 9490 | $\mathbf{3 3 9 4}$ | $f_{23}$ | 28429 | 18816 | 5873 | 9134 | $\mathbf{4 4 3 9}$ |
| $f_{12}$ | 88007 | 100336 | 66012 | 70527 | $\mathbf{5 5 6 5 1}$ | $f_{24}$ | 72011 | 71508 | 38101 | 60152 | $\mathbf{2 5 0 7 1}$ |

Bold values indicate the best value achieved among all algorithms

Table 4 Mean Error (ME) achieved by the algorithms in 100 runs, TP: Test Problems

| TP | ABC | BSFABC | GABC | MABC | ABCGLN | TP | ABC | BSFABC | GABC | MABC | ABCGLN |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f_{1}$ | $8.17 \mathrm{E}-06$ | $7.49 \mathrm{E}-06$ | $8.26 \mathrm{E}-06$ | $8.95 \mathrm{E}-06$ | $\mathbf{7 . 4 6 E}-\mathbf{0 6}$ | $f_{13}$ | $9.34 \mathrm{E}+00$ | $4.12 \mathrm{E}+00$ | $8.51 \mathrm{E}+00$ | $1.06 \mathrm{E}+00$ | $\mathbf{1 . 0 2 E}-\mathbf{0 1}$ |
| $f_{2}$ | $2.28 \mathrm{E}-04$ | $5.96 \mathrm{E}-06$ | $6.92 \mathrm{E}-06$ | $9.24 \mathrm{E}-06$ | $\mathbf{3 . 2 9 E}-\mathbf{0 6}$ | $f_{14}$ | $7.08 \mathrm{E}-06$ | $\mathbf{6 . 9 9 E}-\mathbf{0 6}$ | $7.91 \mathrm{E}-06$ | $9.27 \mathrm{E}-06$ | $7.98 \mathrm{E}-06$ |
| $f_{3}$ | $\mathbf{1 . 6 0 E}+\mathbf{0 0}$ | $1.97 \mathrm{E}+00$ | $2.75 \mathrm{E}+00$ | $3.60 \mathrm{E}+01$ | $3.85 \mathrm{E}+00$ | $f_{15}$ | $8.32 \mathrm{E}-06$ | $2.05 \mathrm{E}-05$ | $5.64 \mathrm{E}-06$ | $5.23 \mathrm{E}-06$ | $\mathbf{5 . 0 2 E}-\mathbf{0 6}$ |
| $f_{4}$ | $8.28 \mathrm{E}-06$ | $8.11 \mathrm{E}-06$ | $9.06 \mathrm{E}-06$ | $9.51 \mathrm{E}-06$ | $\mathbf{8 . 0 6 E}-\mathbf{0 6}$ | $f_{16}$ | $1.46 \mathrm{E}-01$ | $3.07 \mathrm{E}-02$ | $2.44 \mathrm{E}-02$ | $1.25 \mathrm{E}-02$ | $\mathbf{7 . 3 7 E}-\mathbf{0 3}$ |
| $f_{5}$ | $7.22 \mathrm{E}-06$ | $7.01 \mathrm{E}-06$ | $7.98 \mathrm{E}-06$ | $9.23 \mathrm{E}-06$ | $\mathbf{6 . 9 3 E}-\mathbf{0 6}$ | $f_{17}$ | $1.77 \mathrm{E}-04$ | $1.34 \mathrm{E}-04$ | $9.92 \mathrm{E}-05$ | $1.95 \mathrm{E}-04$ | $\mathbf{1 . 2 8 E}-\mathbf{0 5}$ |
| $f_{6}$ | $7.46 \mathrm{E}-06$ | $7.33 \mathrm{E}-06$ | $8.31 \mathrm{E}-06$ | $9.11 \mathrm{E}-06$ | $\mathbf{7 . 0 4 E}-\mathbf{0 6}$ | $f_{18}$ | $8.14 \mathrm{E}+00$ | $2.72 \mathrm{E}+00$ | $5.59 \mathrm{E}-01$ | $8.54 \mathrm{E}-01$ | $\mathbf{8 . 4 5 E}-\mathbf{0 2}$ |
| $f_{7}$ | $9.89 \mathrm{E}+01$ | $8.67 \mathrm{E}+01$ | $1.06 \mathrm{E}+02$ | $1.47 \mathrm{E}+00$ | $\mathbf{9 . 3 5 E}-\mathbf{0 3}$ | $f_{19}$ | $6.98 \mathrm{E}-06$ | $7.09 \mathrm{E}-06$ | $7.16 \mathrm{E}-06$ | $8.34 \mathrm{E}-06$ | $\mathbf{6 . 7 0 E}-\mathbf{0 6}$ |
| $f_{8}$ | $7.63 \mathrm{E}-06$ | $\mathbf{7 . 1 5 E}-\mathbf{0 6}$ | $8.17 \mathrm{E}-06$ | $9.02 \mathrm{E}-06$ | $8.88 \mathrm{E}-06$ | $f_{20}$ | $7.44 \mathrm{E}-04$ | $4.04 \mathrm{E}-03$ | $1.54 \mathrm{E}-04$ | $6.22 \mathrm{E}-04$ | $\mathbf{1 . 5 3 E}-\mathbf{0 4}$ |
| $f_{9}$ | $9.05 \mathrm{E}-06$ | $8.88 \mathrm{E}-06$ | $9.22 \mathrm{E}-06$ | $9.49 \mathrm{E}-06$ | $\mathbf{8 . 4 0 E}-\mathbf{0 6}$ | $f_{21}$ | $\mathbf{7 . 6 3 E}-\mathbf{0 6}$ | $8.16 \mathrm{E}-06$ | $8.32 \mathrm{E}-06$ | $9.06 \mathrm{E}-06$ | $8.89 \mathrm{E}-06$ |
| $f_{10}$ | $9.77 \mathrm{E}-01$ | $9.60 \mathrm{E}-01$ | $9.34 \mathrm{E}-01$ | $9.33 \mathrm{E}-01$ | $\mathbf{8 . 0 9 E}-\mathbf{0 1}$ | $f_{22}$ | $4.76 \mathrm{E}-07$ | $4.45 \mathrm{E}-07$ | $4.80 \mathrm{E}-07$ | $4.67 \mathrm{E}-07$ | $\mathbf{1 . 7 1 E}-\mathbf{1 4}$ |
| $f_{11}$ | $\mathbf{5 . 4 9 E - \mathbf { 0 6 }}$ | $6.08 \mathrm{E}-06$ | $5.50 \mathrm{E}-06$ | $7.57 \mathrm{E}-06$ | $5.96 \mathrm{E}-06$ | $f_{23}$ | $1.95 \mathrm{E}-03$ | $1.96 \mathrm{E}-03$ | $1.95 \mathrm{E}-03$ | $1.95 \mathrm{E}-03$ | $\mathbf{1 . 9 4 E}-\mathbf{0 3}$ |
| $f_{12}$ | $3.25 \mathrm{E}-02$ | $6.90 \mathrm{E}-02$ | $7.39 \mathrm{E}-06$ | $8.34 \mathrm{E}-06$ | $\mathbf{7 . 0 5 E}-\mathbf{0 6}$ | $f_{24}$ | $9.22 \mathrm{E}-16$ | $7.29 \mathrm{E}-16$ | $9.36 \mathrm{E}-16$ | $9.23 \mathrm{E}-16$ | $\mathbf{7 . 1 8 E}-\mathbf{1 6}$ |

Bold values indicate the best value achieved among all algorithms


Fig. 3 Boxplots graph for considered algorithms on test problems $f_{1}-f_{24}$

Table 5 Comparison of algorithms based on the MannWhitney U rank sum test at a $\alpha=0.05$ significance level and mean number of function evaluations, TP: Test Problem

| TP | Mann-Whitney U rank sum test with ABCGLN |  |  |  | TP | Mann-Whitney U rank sum test with ABCGLN |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ABC | BSFABC | GABC | MABC |  | ABC | BSFABC | GABC | MABC |
| $f_{1}$ | + | $+$ | $+$ | $+$ | $f_{13}$ | $+$ | + | $+$ | + |
| $f_{2}$ | $+$ | + | + | + | $f_{14}$ | $+$ | + | $+$ | + |
| $f_{3}$ | - | - | - | + | $f_{15}$ | $+$ | + | + | + |
| $f_{4}$ | + | + | + | + | $f_{16}$ | + | + | + | + |
| $f_{5}$ | + | + | + | + | $f_{17}$ | $+$ | + | + | + |
| $f_{6}$ | $+$ | $+$ | $+$ | $+$ | $f_{18}$ | $+$ | $+$ | $+$ | + |
| $f_{7}$ | $+$ | $+$ | $+$ | $+$ | $f_{19}$ | $+$ | $+$ | $+$ | $=$ |
| $f_{8}$ | + | + | + | + | $f_{20}$ | $+$ | + | + | + |
| $f_{9}$ | $+$ | $+$ | $+$ | $+$ | $f_{21}$ | $+$ | $+$ | $+$ | + |
| $f_{10}$ | $+$ | + | + | + | $f_{22}$ | $+$ | + | + | + |
| $f_{11}$ | + | + | + | + | $f_{23}$ | $+$ | + | $=$ | + |
| $f_{12}$ | $+$ | $+$ | $+$ | $+$ | $f_{24}$ | $+$ | $+$ | $+$ | $+$ |

variable varies from 0 to 1 as given in Bansal and Sharma (2012). Following are the resultant cases:

1. $k_{1}=W, k_{2}=k_{3}=\frac{1-W}{2}, 0 \leq W \leq 1$;
2. $k_{2}=W, k_{1}=k_{3}=\frac{1-W}{2}, 0 \leq W \leq 1$;
3. $k_{3}=W, k_{1}=k_{2}=\frac{1-W}{2}, 0 \leq W \leq 1$

The PI graphs corresponding to above cases (1), (2) and (3) for all the algorithms are shown in Fig. 4(a),(b) and (c) respectively where horizontal axis represents the weights $k_{1}, k_{2}$ and $k_{3}$ and vertical axis represents the PI. In case (1), equal weights are given to AFE and ME while weight to SR varies from 0 to 1 . In case (2), SR and ME are assigned equal weights and weight to AFE varies in range 0 to 1 , while in case (3), equal weights are assigned to SR and AFE and weight to ME varies in range $(0,1)$. It can be observed from Fig. 4 that PI of ABCGLN is much higher than all the considered algorithms for all three cases.

Further, to compare convergence speed of ABCGLN with respect to other algorithms, we calculated the
acceleration rate (AR) based on the AFEs, which is defined below in Eq. (6). A smaller AFEs means higher convergence speed. To minimize the effect of algorithm's stochastic nature, we averaged the reported function evaluations over 100 runs for each test problem.
$A R=\frac{A F E_{A L G O}}{A F E_{A B C G L N}}$,
where, ALGO is any of the algorithms ABC, BSFABC, GABC and MABC. Table 6 shows a comparison between ABCGLN and other considered recent modified ABC algorithms in terms of $A R$. Here, $A R>1$ means ABCGLN is faster i.e, in less number of function evaluations it achieved the required optimum with acceptable error. It is clear from the Table 6 that convergence speed of ABCGLN is much better than the other considered algorithms for all the test problems except functions $f_{3}$ as it took on an average half number of function evaluations as compare to other algorithms.


Fig. 4 Performance index for algorithms on test problems; (a) for case (1), (b) for case (2) and (c) for case (3)

Table 6 Acceleration Rate (AR) of ABCGLN as compared to other considered algorithms , TP: Test Problems

| TP | ABCGLN vs |  |  |  | TP | ABCGLN vs |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ABC | BSFABC | GABC | MABC |  | ABC | BSFABC | GABC | MABC |
| $f_{1}$ | 2.152 | 3.114 | 2.600 | 2.398 | $f_{13}$ | 1.403 | 1.370 | 1.313 | 1.200 |
| $f_{2}$ | 1.900 | 1.781 | 1.400 | 1.769 | $f_{14}$ | 2.883 | 2.194 | 2.667 | 2.090 |
| $f_{3}$ | 0.980 | 0.966 | 0.941 | 1.015 | $f_{15}$ | 4.445 | 4.398 | 2.207 | 1.704 |
| $f_{4}$ | 2.721 | 2.376 | 2.652 | 2.418 | $f_{16}$ | 2.568 | 2.053 | 2.062 | 1.800 |
| $f_{5}$ | 2.310 | 3.191 | 2.868 | 2.285 | $f_{17}$ | 3.028 | 2.400 | 1.674 | 2.879 |
| $f_{6}$ | 2.300 | 2.294 | 3.085 | 2.228 | $f_{18}$ | 2.255 | 2.127 | 1.367 | 1.875 |
| $f_{7}$ | 1.524 | 1.524 | 1.524 | 1.524 | $f_{19}$ | 1.523 | 2.010 | 1.456 | 1.427 |
| $f_{8}$ | 2.060 | 3.046 | 2.503 | 2.227 | $f_{20}$ | 3.216 | 3.590 | 1.390 | 2.941 |
| $f_{9}$ | 2.822 | 2.233 | 2.815 | 1.756 | $f_{21}$ | 2.219 | 3.914 | 2.018 | 1.786 |
| $f_{10}$ | 16.785 | 13.917 | 10.817 | 2.322 | $f_{22}$ | 14.581 | 10.860 | 11.132 | 11.268 |
| $f_{11}$ | 4.742 | 4.316 | 4.425 | 2.796 | $f_{23}$ | 6.404 | 4.239 | 1.323 | 2.058 |
| $f_{12}$ | 1.581 | 1.803 | 1.186 | 1.267 | $f_{24}$ | 2.872 | 2.852 | 1.520 | 2.399 |

## 5 Conclusion

In this article, local and global neighborhood based position update strategy is incorporated in employed bee phase of ABC. In the proposed strategy, each employed bee gets updated using best solutions in its local and global neighborhoods as well as random members from these neighborhoods. Here, local neighborhood of any solution means the sub population around that bee based on index not based on search space or objective space while global neighborhood is the entire swarm for each solution. Therefore the exploration (due to presence of local neighborhoods) and exploitation (due to presence of global neighborhood) can be well balanced. The performance of the proposed algorithm ABCGLN has been extensively compared with other recent variants of ABC namely, GABC, BSFABC and MABC. Through the extensive and statistical experiments, it can be stated that the proposed strategy uplifted the ABC algorithm's performance in terms of robustness, accuracy and efficiency and hence ABCGLN can be considered as a promising alternative algorithm to solve a variety of optimization problems. In
future, authors will apply proposed algorithm to solve some real world optimization problems.

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[^1]:    $D$ dimensions, $C$ characteristic, $U$ unimodal, $M$ multimodal, $S$ separable, $N$ non-separable, $A E$ acceptable error

