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An improved linear prediction evolution algorithm based on nonlinear least square fitting model for optimization

A. M. Mohiuddin · Jagdish Chand Bansal *

Abstract The linear prediction evolution algorithm (LPE) is a recent addition in the field of optimization algorithm with few parameters and high exploration capability. LPE has shown excellent results in some real-world problems but still suffers from some issues such as slow convergence speed. To improve the performance of the LPE algorithm, this study presents an improved linear prediction evolution algorithm (ILPE) to enhance its exploration capability. The proposed ILPE algorithm treats the population series of evolutionary algorithms as a time series and uses the non-linear least square fitting model as a reproduction operator to forecast the next generation. The performance of the proposed ILPE is verified using the CEC2014 and CEC2017 benchmark functions. The comparison results show that ILPE outperforms LPE, and it is highly competitive with other state-of-the-art optimization algorithms.

Keywords Linear prediction evolution algorithm, Least square fitting, Meta-heuristics, Nonlinear Optimization

1 Introduction

Real world problems form interdisciplinary areas, usually including non-linear, discontinuous, and non-convex objective functions with high-dimensional and complex boundary conditions. To get satisfactory results, deterministic mathematical methods are often infeasible to apply. Meta-heuristic algorithms have received much attention as powerful and general optimization methods for solving various complex problems (Dulebenets, 2020; Bansal & Farswan, 2017). Many new metaheuristic algorithms have been developed in the last few decades, including swarm intelligence-based algorithms such as ant colony optimization (ACO)(Dorigo, Birattari, & Stutzle, 2006), particle swarm optimization (PSO)(Kennedy & Eberhart, 1995), artificial bee colony algorithm (ABC)(Karaboga & Basturk, 2007), grey wolf optimization (GWO)(Mirjalili, Mirjalili, & Lewis, 2014), teaching learning-based optimization (TLBO)(Rao, Savsani, & Vakharia, 2011), spider monkey optimization (SMO)(Bansal, Sharma, Jadon, & Cle

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2014) and cuckoo search optimization (CSO)(Gandomi, Yang, & Alavi, 2013), etc. Another category of meta-heuristic algorithms are the evolutionary algorithms. These are based on the biological laws of evolution. Few popular evolutionary algorithms are differential evolution (DE)(Storn & Price, 1997), genetic algorithm (GA)(Holland et al., 1992), genetic programming (GP)(Koza & Koza, 1992) etc.

Prediction based evolutionary algorithm is the new trend in the field of evolutionary algorithms. The Grey prediction evolution algorithm (GPE) (Hu, Xu, Su, Zhu, & Guo, 2020), is the first prediction-based evolutionary algorithm that incorporates the principles of grey prediction theory. In the initial stages of its development, the algorithm utilized the even grey model (EGM (1,1)) (Liu, Zeng, Liu, Xie, & Yang, 2015), as its foundation model to create a new reproduction operator. Especially, the GPE algorithm employs the predicted values produced by EGM (1,1) model as the evolutionary offspring. The distinguishing feature of the GPE algorithm is its egm11 reproduction operator (Hu, Xu, et al., 2020), which is based on even grey model. The performance of the algorithm has also been analyzed using benchmark functions from CEC 2005 and some constrained engineering design problems. Hu et al. proposed a grey prediction evolution algorithm based on even difference grey model (GPEed) (Hu, Gao, & Su, 2021). The proposed GPEed developed a reproduction operator based on even difference grey model to predict the next generation of populations, which is the core innovation of GPEed. In order to accelerate the performance of the GPE algorithm Gao, Hu et al. used an acceleration term to improve the accuracy of the prediction and proposed grey prediction evolution algorithm based on accelerated even grey model (GPEae) (Gao, Hu, Xiong, & Su, 2020). Dai et al. proposed topological opposition-based learning to enhance the local search capability of GPE (TOGPE) (Dai, Hu, Li, Xiong, & Su, 2020) algorithms and successfully applied to solve CEC 2005, and CEC 2014 benchmark problems. Several variants of GPE have recently been developed to achieve adaptiveness in GPE and tackle a range of optimization problems, such as multimodal multi-objective problems, economic dispatch problems, and constrained engineering design problems ((Xu, Hu, Su, Li, & Dai, 2020), (Xu et al., 2020), (Hu, Li, et al., 2020), (Gao, Hu, Miao, Zhang, & Su, 2022), (Xiang, Su, Huang, & Hu, 2022)).

In contrast to other evolutionary algorithms, the LPE proposed in 2021 (Gao, Hu, & Tong, 2021), is inspired by the linear least square fitting model(York, 1966). It is a new metaheuristic algorithm inspired by a linear mathematical model that treats a population series as a time series and predicts offspring using a linear least-squares fitting model. It has simple code features, only one parameter (population size), and potent exploration capabilities. This has been successfully applied to 10-dimensional CEC2014, CEC2017 benchmark functions (Liang, Qu, & Suganthan, 2013; G. Wu, Mallipeddi, & Suganthan, 2017), and seven engineering design problems such as three-bar truss design, pressure vessel design, tension/compression spring design, welded beam design, tabular column design, gear train design, and cantilever beam design problems.

LPE is proposed based on the most widely used static model, the linear least square fitting model, which considers the population series as a time series and approximates a linear move that may exist in the subsequent generation. This makes LPE to use the least-squares linear fitting model as the reproducibility operator to generate the next generation. The essence of this technique is an iterative predictive method in the evolutionary framework and an assurance of stability in evolution. Though LPE behaves well on 10-dimensional CEC2014 and CEC2017 benchmark problems in solution accuracy, the robustness and convergence rate were not fair enough (Gao et al., 2021). Also, for the 30-dimensional problem, it suffers from showing its effectiveness in terms of convergence rate, solution accuracy and robustness compared to the other state-of-the-art algorithms. Because the linear strategy fails to explore the whole search space efficiently and after some iterations, it stagnates with no further explorations.

In this paper, an improved linear prediction evolution algorithm (ILPE) is introduced to overcome all such issues. Same as LPE, the ILPE considers the population series as a time series but uses a non-linear least-square fitting technique as a reproduction operator to predict the next-generation population. Also, to overcome local minima stagnation it uses a random perturbation if the three-generation populations are too close. In order to validate the superiority of ILPE, we use 30-dimensional CEC2014 and CEC2017 benchmark functions.

The rest of the paper is organized as follows. Section 2 briefly discusses the preliminaries including linear prediction evolution algorithm (LPE). In section 3, a detailed description of the proposed improved linear prediction evolution algorithm is presented. In section 4, the detailed experimental result and different statistical analyses are presented. Finally concluding remarks are given in section 5.

Table 1: List of abbreviations and notations

Abbreviations/ Notations	Description
LPE	Linear prediction evolution algorithm
ACO	Ant colony optimization
PSO	Particle swarm optimization
ABC	Artificial bee colony algorithm
GWO	Grey wolf optimization
TLBO	Teaching learning based optimization
SMO	Spider monkey optimization
CSO	Cuckoo search optimization
DE	Differential evolution
GA	Genetic algorithm
GP	Genetic programming
GPE	Grey prediction evolution algorithm
GPEed	Grey prediction evolution algorithm based on even difference grey model
EGM	Even grey model
MGPE	Multivariable grey prediction evolution algorithm
TOGPE	Grey prediction evolution algorithm based on topological opposition based learning
MMGPE	Grey prediction evolution algorithm for multimodal multiobjective optimization
GPEAae	Grey prediction evolution algorithm based on accelerated even grey model
NeGPE	A simplified non-equidistant GPE algorithm for global optimization
aGPE	Four adaptive GPE algorithms with different types of parameters setting techniques
ILPE	Improved linear prediction evolution algorithm
low	Lower bounds
up	Upper bounds
X^0, X^1 and X^2	First, second and third population
lfp	Linear fitting operator
N	Number of population
D	Dimension of the search space
g_{max}	Maximum number of generations
BEST	Best solution
MEAN	Mean values
STD	Standard deviation values
F	Mutation rate
CR	Crossover rate
C_1	Personal learning coefficient
C_2	Global learning coefficient
P_c	Crossover rate
P_m	Mutation rate
δ	Difference threshold

2 Preliminaries

Linear prediction Algorithm:

The linear prediction evolution algorithm developed by Cong Gao et al. in 2020 (Gao et al., 2021) based on the prediction-based linear least-squares fitting model (Miller, 2006). LPE is an iterative predictive model. It has no crossover operator and the only control parameter is the population size. Also, the LPE reproduction operator is developed from the simplest statistical model, the linear least-squares fitting model. The LPE determines the approximate linear trend that may exist in a subsequent population to achieve iterative optimization.

2.1 Mathematical Model

In this subsection, the linear least square fitting model, and linear prediction operator of LPE are explained.

Linear Least Square Fitting Model(Weisstein, 2002): Assume that $(t_1, x_1), (t_2, x_2), \dots, (t_n, x_n)$ are n input data points. The linear least square model can be represented as

$$x_i = a_0 + a_1 t_i + E_i, \quad i = 1, 2, \dots, n. \quad (1)$$

Here, a_0 and a_1 are coefficients to be determined and E_i represents the error of the i^{th} pair of data. Note that the error E_i is a function of two variables a_0 and a_1 . More details can be found in (Yan & Su, 2009).

Estimation of a_0 and a_1 (Freund, Wilson, & Sa, 2006) : The method of least squares uses to determine a_0 and a_1 . We estimate a_0 and a_1 so that the sum of the squares of the differences between the observations x_i and the straight line is a minimum. Thus the least square criterion is

$$S = \sum_{i=1}^n E_i^2 = \sum_{i=1}^n (x_i - a_0 - a_1 t_i)^2 \quad (2)$$

The above equation is called the square loss function. The objective is to find values for a_0 and a_1 so that error is minimum. From multi-variable calculus we learn that this requires us to find the values of $(a_0; a_1)$ such that the gradient of S with respect to our variables (which are a_0 and a_1) vanishes. To get the value of the undetermined coefficients a_0 and a_1 , partial derivatives of S with respect to a_0 and a_1 are obtained.

$$\begin{cases} \frac{\partial S}{\partial a_0} = 2 * \sum_{i=1}^n (x_i - a_0 - a_1 t_i)(-1) = 0 \\ \frac{\partial S}{\partial a_1} = 2 * \sum_{i=1}^n (x_i - a_0 - a_1 t_i)(-t_i) = 0 \end{cases} \quad (3)$$

The value of a_0 and a_1 can be obtained from the following matrix

$$\begin{bmatrix} \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i t_i \end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^n t_i \\ \sum_{i=1}^n t_i & \sum_{i=1}^n t_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \quad (4)$$

Now we construct a linear least-squares model using the three input data (x_1, x_2, x_3) to produce the fourth predicted data \hat{x}_4 . Consider that the four related data points are equidistant and specified as $(1, x_1), (2, x_2), (3, x_3)$ and $(4, \hat{x}_4)$, respectively. From equation (4), we get the value of a_0 and a_1 .

$$\left. \begin{array}{l} a_0 = \frac{1}{2}x_3 - \frac{1}{2}x_1 \\ a_1 = \frac{4}{3}x_1 + \frac{1}{3}x_2 - \frac{2}{3}x_3 \end{array} \right\} \quad (5)$$

Set $E_i = 0$ and putting the value of a_0 and a_1 in equation (1) the fourth predicted data will be

$$\hat{x}_4 = \frac{4}{3}x_3 + \frac{1}{3}x_2 - \frac{2}{3}x_1 \quad (6)$$

2.2 Initialization

The first step of LPE is to randomly initialize a population containing $3N$ individuals in the D dimensional search space. We denote each population having N individuals by $\mathbf{x}_i^g = (x_{i,1}^g, x_{i,2}^g, \dots, x_{i,D}^g)$ here, $i = 1, 2, \dots, N$ and $g = 0, 1, 2, \dots, g_{max}$, where g is the generation and g_{max} is the maximum number of generations. The i^{th} individuals in the j^{th} dimensions are initialized by using the following equation.

$$x_{i,j}^g = low_j + r.(up_j - low_j) \quad (7)$$

Where r represents, the uniformly distributed random number between 0 and 1. low_j , up_j are lower and upper bounds of the j^{th} variable, respectively. Next, we divide the $3N$ individual into three populations, and sort according to the objective function value of the individual. The top N individuals are considered as the first-population $X^0(g = 0)$. At the same time, the middle N individuals are considered as the second population $X^1(g = 1)$, and the last N individuals are considered as the third population $X^2(g = 2)$. These three populations form the initial population series as a time series for predicting the next generation of populations.

2.3 Reproduction

Based on linear least square fitting technique a new reproduction operator called the linear fitting prediction operator (lfp operator) (Gao et al., 2021) has been developed. The lfp operator is defined as follows;

Let X^{g-2} , X^{g-1} , and X^g , ($g \geq 2$) represent three consecutive population series and three individuals $\mathbf{x}_{r_1}^{g-2}$, $\mathbf{x}_{r_2}^{g-1}$, and $\mathbf{x}_{r_3}^g$ are randomly chosen from X^{g-2} , X^{g-1} , and X^g , respectively. Assume that $\mathbf{u}_i^g = (u_{i,1}^g, u_{i,2}^g, \dots, u_{i,D}^g)$, $i = 1, 2, \dots, N$ be the trial vector. Then according to the equation (6) the lfp operator is

$$u_{i,j}^g = \frac{4}{3}x_{r_3,j}^g + \frac{1}{3}x_{r_2,j}^{g-1} - \frac{2}{3}x_{r_1,j}^{g-2} \quad (8)$$

The individual series denoted by $x_{r_3,j}^{g-2}$, $x_{r_2,j}^{g-1}$, $x_{r_1,j}^g$ ($j = 1, 2, \dots, D$) represent the j^{th} dimension of the three individuals $\mathbf{x}_{r_3}^{g-2}$, $\mathbf{x}_{r_2}^{g-1}$, and $\mathbf{x}_{r_1}^g$, respectively.

2.4 Selection Operator

In this phase of LPE, an individual with a better fitness score is selected from the trial population \mathbf{u}_i^g and the target population \mathbf{x}_i^g according to the greedy selection strategy used to update the population. If the objective function f is to be minimized the newly generated populations \mathbf{u}_i^g will pass to the next iteration or not; this will decide by the the greedy approach is described below:

$$x_i^{g+1} = \begin{cases} u_i^g, & \text{if } f(u_i^g) < f(x_i^g) \\ x_i^g, & \text{otherwise} \end{cases} \quad (9)$$

Algorithm 1 Pseudocode of LPE

Input: $N, D, g_{max}, low_j, up_j$
Output: Optimal value of the objective function f.
Initialization: Initialize X^0, X^1 and X^2 using the equation (7).
The three populations form the series as $\text{PopX}=\{X^0, X^1, X^2\}$, respectively.
Linear Prediction:

```

for  $g = 3 : g_{max}$  do
    for  $i = 1 : N$  do
         $l = \text{randperm}(N); m = \text{randperm}(N); n = \text{randperm}(N)$ 
        for  $j = 1 : D$  do
             $X_p = [\text{PopX}\{1,1\}(l(i),j), \text{PopX}\{1,2\}(m(i),j), \text{PopX}\{1,3\}(n(i),j)]$ 
             $u_{i,j}^g = \frac{4}{3}X_p(3) + \frac{1}{3}X_p(2) - \frac{2}{3}X_p(1)$ 
        end for
    end for
    Selection
    for  $i = 1 : N$  do
        if  $f(u_i^g) < f(x_i^g)$  then
             $x_i^{g+1} = u_i^g$ 
        else
             $x_i^{g+1} = x_i^g$ 
        end if
    end for
     $\text{PopX}(1,4) = \{x^{g+1}\}$ 
     $\text{PopX} = \text{PopX}(2,4)$ 
end for

```

3 Proposed Method

3.1 Motivation

The Original LPE algorithm has no parameter except population size to perform the search, and is easy to implement. It has no mutation and crossover operator like other evolutionary algorithms. However, experimental analysis has shown that LPE is vulnerable to sticking to local optima in some cases due to the inadequate ability to search of linear least-squares fitting models. Though it is a good optimizer for lower-dimensional problems in terms of solution convergence rate and accuracy, its robustness is competitive (Gao et al., 2021).

For higher-dimensional problems, LPE fails to show its accuracy and convergence as compared to other metaheuristic algorithms. Therefore, there is a scope to improve the ability to search of linear models and make them better optimizer. So, we have introduced a nonlinear least square fitting model in place of linear fitting operator to predict the next generation of population so that it can explore the large area of the search space. Thus, by using the non-linear least square fitting model, the main aim of this paper is to increase the LPE algorithm's exploration capability and rate of convergence.

3.2 Mathematical Model:

In this sub-section the details of the nonlinear least square fitting model is described as follows:

Non-linear least square fitting model (Mullineux, 2008; H. Wu, 2002): Assume that $(t_1, x_1), (t_2, x_2), \dots, (t_n, x_n)$ are n input data points, the non-linear least square fitting model can be

expressed as

$$x_i = a_0 + a_1 \sin(\omega * t_i) + a_2 \cos(\omega * t_i) + E_i \quad (10)$$

Where a_0 , a_1 and a_2 are coefficients to be determined and E_i represents the residuals of i^{th} pair of the data. And ω lies between $(0, \pi)$.

Estimation of the constants: We use the least square method to estimate a_0 , a_1 and a_2 . Thus the least square criterion is

$$S = \sum_{i=1}^n E_i^2 = \sum_{i=1}^n (x_i - a_0 - a_1 \sin(\omega * t_i) - a_2 \cos(\omega * t_i))^2 \quad (11)$$

The above equation is also called the sum of the square loss. Our aim is to find the value of a_0 , a_1 and a_2 to minimize the error. To find the undermined coefficients a_0 , a_1 and a_2 we differentiate the function S partially with respect to a_0 , a_1 and a_2 . The resulting equations are as follows.

$$\begin{cases} \frac{\partial S}{\partial a_0} = 2 * \sum_{i=1}^n (x_i - a_0 - a_1 \sin(\omega * t_i) - a_2 \cos(\omega * t_i))(-1) = 0 \\ \frac{\partial S}{\partial a_1} = 2 * \sum_{i=1}^n (x_i - a_0 - a_1 \sin(\omega * t_i) - a_2 \cos(\omega * t_i))(-\sin(\omega t_i)) = 0 \\ \frac{\partial S}{\partial a_2} = 2 * \sum_{i=1}^n (x_i - a_0 - a_1 \sin(\omega * t_i) - a_2 \cos(\omega * t_i))(-\cos(\omega t_i)) = 0 \end{cases} \quad (12)$$

The value of a_0 , a_1 and a_2 can be obtained from the following matrix

$$\begin{bmatrix} \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i \sin(\omega t_i) \\ \sum_{i=1}^n x_i \cos(\omega t_i) \end{bmatrix} = \begin{bmatrix} n & \sum_{i=1}^n \sin(\omega t_i) & \sum_{i=1}^n \cos(\omega t_i) \\ \sum_{i=1}^n \sin(\omega t_i) & \sum_{i=1}^n \sin^2(\omega t_i) & \frac{\sum_{i=1}^n \sin(2\omega t_i)}{2} \\ \sum_{i=1}^n \cos(\omega t_i) & \frac{\sum_{i=1}^n \sin(2\omega t_i)}{2} & \sum_{i=1}^n \cos^2(\omega t_i) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \quad (13)$$

Now, similar to LPE let the four equally spaced data points are $(1, x_1)$, $(2, x_2)$, $(3, x_3)$ and $(4, \hat{x}_4)$, respectively. The values of a_0 , a_1 , and a_2 , depends upon the selection of ω . Then with different values of $\omega \in (0, \pi)$ with $\pi/6$ step size, we get the fourth predictive data point using equation (13). With $E_i = 0$, the models corresponding to the different values of ω are shown in Table 1.

Table 2: Non-linear least square fitting models

Name	value of ω	models
model-1	$\pi/6$	$\hat{x}_4 = 0.99x_1 - 2.74x_2 + 2.74x_3$
model-2	$\pi/4$	$\hat{x}_4 = x_1 - 2.41x_2 + 2.41x_3$
model-3	$\pi/3$	$\hat{x}_4 = x_1 - 2x_2 + 2x_3$
model-4	$\pi/2$	$\hat{x}_4 = x_1 - x_2 + x_3$
model-5	$2\pi/3$	$\hat{x}_4 = x_1$
model-6	$5\pi/6$	$\hat{x}_4 = x_1 + 0.73x_2 - 0.73x_3$

3.3 Model Selection:

Various model with different values of ω has been evaluated over CEC2014 benchmark functions (Liang et al., 2013). Here we consider 30-dimensional CEC2014 benchmark functions, and the best values (BEST), mean values (MEAN), and standard deviation values (STD) are recorded over 51 independent runs and are presented in Table 4. The best value for each indicator is in bold.

Table 3: Parameter of each algorithm

Algorithm	Parameters
DE/rand/1	$F=CR=0.5$
PSO	$C_1 = C_2 = 1.4; w=0.7$
GPE	$\delta = 0.01;$
GA	$P_c=0.8, P_m=0.3$
ILPE	$\delta = 0.06$

Table 4 shows the results of all 30-benchmark functions of CEC2014 benchmarks set in the 30-dimensional search space, where ‘M’ denotes the model and ‘BP’ denotes the benchmark problems. Analysis of the obtained results shows that model-6 performs better than other models in the best values with an average ranking of 1.4, while the average ranking of model-1, model-2, model-3, model-4, and model-5 are 2.3, 4.06, 4.33, 2.9, and 6. The average rank of mean and standard deviation values for model-6 are 1.43 and 1.46, which is lower than the other models. Table 4 shows the average ranks of all models. From Table 3 we can see that model-6 has the smallest BEST and MEAN function values for all functions except F_{10} , F_{11} , F_{16} , F_{22} , F_{26} , and F_{5} , F_{10} , F_{11} , F_{16} , F_{27} . Model-6 also has the lowest standard deviation (STD) values for all functions except F_5 , F_6 , F_{11} , F_{12} , F_{16} , F_{22} , F_{24} , F_{25} , and F_{27} . With the above analysis, we choose model-6 as non-linear least square fitting model.

3.4 Pseudocode

As mentioned above, the original LPE has no parameters other than population size. The proposed ILPE uses a nonlinear least-squares fitting model to achieve a better balance between exploration and exploitation. Similar to the original LPE after three population initialization (equation (7)), the ILPE generates the offspring’s by using the reproduction operator (model-6) and selection operator (formula (9)). As compared to LPE, the ILPE uses the non-linear least square fitting model as a reproduction operator. If all values of the data series are equal, the ILPE uses random perturbation to generate the subsequent value to avoid the local minima stagnations. Otherwise, ILPE reproduction operator is used to predict the next value. If the new solution exceeds the feasible region, then it is replaced by the random solution in the feasible space.

4 Numerical experiment

To numerically validate the performance of the proposed ILPE algorithm, this study uses the CEC2014 (Liang et al., 2013) and CEC2017 (G. Wu et al., 2017) benchmark functions. The CEC2014 and CEC2017 benchmark functions are used to compare ILPE with original LPE and nine other state of the art meta-heuristics algorithms including GPE, GPEae, GPEed, DE, PSO, ABC, GWO, GA, and TLBO algorithms. All the functions of CEC2014 and CEC2017 are divided into four categories that is uni-modal, multi-modal, hybrid and composite functions.

4.1 Parameter setting

The parameter setting of all the considered algorithms is shown in Table 3. Each algorithm runs independently 51 times with a population size of 50. The maximum number of iterations is set to 2000 and the dimension of each problem is set to 30.

Algorithm 2 Pseudocode of ILPE

Input: $N, D, g_{max}, low_j, up_j, \delta$
Output: Optimal value of the objective function f.
Initialization: Initialize X^0, X^1 and X^2 using the equation (7).
The three populations form the series as $\text{PopX}=\{X^0, X^1, X^2\}$, respectively.

ILPE Prediction:

```

for  $g = 3 : g_{max}$  do
     $t = 0.01 - (3.99/g_{max}).(g - g_{max})$ ;
    for  $i = 1 : N$  do
         $l = \text{randperm}(N); m = \text{randperm}(N); n = \text{randperm}(N)$ 
        for  $j = 1 : D$  do
             $X_p = [\text{PopX}\{1,1\}(l(i,j), \text{PopX}\{1,2\}(m(i,j), \text{PopX}\{1,3\}(n(i,j)]$ 
            if  $|\min(X_p) - \max(X_p)| < \delta$  then
                 $u_{i,j}^g = X_p(1) + t.|\min(X_p) - \max(X_p)|$ 
            else
                 $u_{i,j}^g = X_p(1) + 0.73X_p(2) - 0.73X_p(3)$ 
            end if
        end for
    end for
end for
Selection
for  $i = 1 : N$  do
    if  $f(u_i^g) < f(x_i^g)$  then
         $x_i^{g+1} = u_i^g$ 
    else
         $x_i^{g+1} = x_i^g$ 
    end if
end for
     $\text{PopX}(1,4) = \{x^{g+1}\}$ 
     $\text{PopX} = \text{PopX}(2,4)$ 
end for

```

4.2 Comparison of ILPE with LPE algorithm on CEC2014 benchmark functions

In this subsection numerical results of ILPE and original LPE are shown in Table 6, including three performance indicators, BEST, MEAN, and STD. The optimum value for each indicator is shown in bold. The comparison result shows that ILPE outperforms LPE as it gives the best result for all the 30 functions except $F26$. ILPE has smallest mean values for all functions except $F22$ and $F23$ and has smallest standard deviation values all functions except $F6, F11, F16, F22, F24, F25, F28$. The average rank among the algorithms, for the best values of ILPE is 1.63 while the average rank for the best values of LPE is 5.9. Also, the average rank among the algorithms, for MEAN and STD values of ILPE is superior than LPE. Table 10 shows the average ranking of the considered algorithm. These results indicate that ILPE's performance is superior in terms of robustness and solution accuracy.

4.3 Comparison of ILPE with LPE algorithm on CEC2017 benchmark functions

Experimental results of ILPE and original LPE are shown in Table 7, including three performance measures, BEST, MEAN, and the STD of the considered algorithms. The best value for each indicator is shown in boldface. Comparison results show that ILPE outperforms LPE as it gives the best result for all functions except $F16, F19$, and $F20$. ILPE has smallest mean values in all functions except $F14, F17$ and $F20$ and has smallest standard deviation in all functions except $F1, F6, F10, F14, F17, F23, F24, F26$ and $F29$. The average ranking among the algorithms for the best values

of ILPE is 1.37, while the average rank for the best values of LPE is 4.1. Also, the average ranks for MEAN and STD values of ILPE are superior to LPE. The average ranking is shown in Table 11. Based on the experimental results of CEC2017 benchmark functions, it can be concluded that ILPE is better in terms of robustness and convergence speed.

4.4 Comparison of ILPE with other meta-heuristic algorithms on CEC2014 benchmark functions

In this subsection, the experimental result of ILPE and other meta-heuristic algorithms are shown in Table 8, based on same performance indicators that is BEST, MEAN and STD values of the algorithms over 51 runs. The best results of each algorithms are highlighted in boldface. As compared to other meta-heuristic algorithms, for best values ILPE ranks first for functions $F_1, F_4, F_5, F_8, F_9, F_{11}, F_{12}, F_{15} - F_{22}, F_{25}, F_{26}, F_{29}$ and F_{30} . The result indicates that ILPE performs very good for hybrid functions and composite functions. ILPE also has smallest mean for the functions $F_1, F_5, F_6, F_8, F_9, F_{11}, F_{12}, F_{14} - F_{21}, F_{29}, F_{30}$ and smallest standard deviation for the functions $F_{12}, F_{18}, F_{20}, F_{21}, F_{29}$ and F_{30} . Also, the average rank of the best values of ILPE is 1.63 while the average rank of the best values of GPE, GPEae, GPEed, PSO, DE, ABC, GWO, GA, and TLBO are 4.2, 4.47, 5.2, 5.9, 5.07, 9.43, 9.77, 7.07, and 7.37, respectively. The average rank of mean values of ILPE 1.60 is superior than those of other considered algorithms. The average rank of ILPE in terms of standard deviation is 3.73 that is worse than only DE, but it is better than the other nine algorithms. Thus, from Table 8 and Table 10, we can conclude that ILPE is a very good optimizer as compared to the GPE, GPEae, GPEed, PSO, ABC, GWO, GA, and TLBO, and shows highly competitive robustness with the competitors.

4.5 Comparison of ILPE with other meta-heuristic algorithms on CEC2017 benchmark function

Like the above numerical experiments, the experimental results of ILPE on CEC2017 benchmark functions with other meta-heuristic algorithms are shown in Table 9. As compared to other meta-heuristic algorithms, for best values ILPE ranks first for functions $F_2, F_4, F_5, F_7, F_8, F_{10} - F_{15}, F_{17}, F_{18}, F_{21}$ and $F_{24} - F_{30}$. These results indicate that ILPE performs very well for complex functions. ILPE also has smallest mean for the functions $F_2, F_4, F_5, F_7, F_8, F_{10} - F_{13}, F_{15}, F_{16}, F_{18}, F_{19}, F_{21}, F_{23}, F_{24}, F_{26}, F_{29}, F_{30}$ and smallest standard deviation values for the functions $F_2, F_{11}, F_{13}, F_{15}, F_{18}$ and F_{19} . The average rank of the best values of ILPE is 1.37, while the average ranks for the best values of GPE, GPEae, GPEed, PSO, DE, ABC, GWO, GA, and TLBO are 3.0, 11, 3.63, 5.6, 4.47, 9.33, 9.17, 7.5 and 6.83, respectively. From the average mean rank, we can see that ILPE rank is 1.43 that is superior to those of other considered algorithms. ILPE ranks superior to GPE, GPEae, GPEed, PSO, ABC, GWO, GA, and TLBO but worse than DE when considering standard deviation. Thus, from the statistical results shown in Table 9 and Table 11, we can see that ILPE performs well as compared to the GPE, GPEae, GPEed, PSO, DE, ABC, GWO, GA, and TLBO.

4.6 Convergence Analysis

A comparison of the convergence process of the proposed ILPE and other considered algorithms can be seen through the convergence curves. The convergence curves of all nine considered algorithms under selected benchmark functions from CEC2014 and CEC2017 benchmark functions are given

in Fig 1 and Fig 2. The functions F1, F4, F8, F13, F15, F17, F18, F19, F20, F25, F29 and F30 has been chosen to represent convergence curves for CEC2014 benchmark functions. Among these functions F1 is unimodal function F4, F8, F13 and F15 are multimodal functions F17, F19, F20 are hybrid functions and F25, F29, F30 are composite functions. Also, the functions F2, F5, F7, F8, F12-F14, F27, F21, F24, F26 and F30 has been selected to describe the convergence curves of CEC2017 benchmark functions. In these functions, F2 is unimodal function F5, F7, F8 are multimodal functions F12-F14, F17 are hybrid functions and F21, F24, F26, and F30 are composite functions respectively. The convergence curves are plotted between iterations and the best value in the intermediate iterations. The objective function values are presented on the vertical axis, and the algorithm's iterations are shown on the horizontal axis in the curves. From these figures, we can see that ILPE not only has a faster convergence rate but also more accurate than its competitors. Based on the above results, it can be concluded that the overall performance of ILPE for CEC2014 and CEC2017 benchmark functions is superior to the other ten competing algorithms.

4.7 Statistical Analysis

In this section, Wilcoxon signed-rank test (Derrac, García, Molina, & Herrera, 2011; Gibbons & Chakraborti, 2020) is used to analyze the significant difference between ILPE and other competitors. The Wilcoxon signed-rank test is a non-parametric test to evaluate whether the difference between the ILPE and other metaheuristic algorithms is sufficiently notable. In this paper, Wilcoxon signed-rank test uses the optimal values (BEST) from Table 8 and Table 9 to compare with the other considered algorithms. This test is performed in pairs with the significance level of 5% using the null hypothesis. If the p-value is less than 0.05, the null hypothesis is rejected. Assume that '+', ' \approx ' and '-' represent that the test result of ILPE performs better, comparable, and inferior to other considered algorithms, respectively. Of the 300 comparisons on the CEC2014 benchmarks set have 273 positive symbols. For CEC2017 benchmark functions, out of 300 comparisons, it shows 283 positive signs. The comparison results are shown in Table 12 and Table 13 with p-value, where 'BP' denotes the benchmark problems. Therefore, from all analyses, it can be concluded that ILPE performs remarkably better as compare to other considered algorithms. The resulting ILPE excels in accuracy, robustness and reliability. This is a definitive improvement on the proposed algorithm.

A more intensive statistical analysis was performed on the numerical result of ILPE, GPE, GPEae, GPEed, LPE, PSO, DE, ABC, GWO, GA, and TLBO. Box plots are empirical distributions of data. For CEC2014 benchmark functions, boxplots for BEST, MEAN, and STD values corresponding to all algorithms ILPE, GPE, GPEae, GPEed, LPE, PSO, DE, ABC, GWO, GA, and TLBO are given in Fig. 3. The boxplot for the CEC2017 benchmark functions are presented in Fig. 4. Boxplot's analysis shows that ILPE is superior to other considered algorithms under consideration for both CEC2014 and CEC2017 benchmark functions.

4.8 Computational Complexity

The computational complexity in these metaheuristic algorithms dictates how long the algorithm requires to calculate the final outcome. The computational complexity of ILPE and LPE algorithms is calculated in terms of big-O notation. The algorithm's computational complexity depends on initialization, fitness evaluation, position update mechanism, mutation, and greedy selection approach. For all the algorithms, the computational complexity of initialization is $O(N \times D)$, fitness evaluation is $O(N)$ time, and the greedy selection approach is $O(N)$. In LPE the prediction mechanism is $O(N \times D)$ time. Therefore, the total computational complexity is $O(N \times D \times M)$; where D is the dimension of the

search space, N is the population number, and M is the maximum number of generations. However, ILPE is based on a non-linear least square fit approach, but the obtained model is linear; therefore similar to the LPE the ILPE prediction process is also $O(N \times D)$ time. So, the total computational complexity of ILPE is $O(N \times D \times M)$. Similarly, GPE computational complexity is also $O(N \times D \times M)$.

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Table 4: Results of non-linear least square fitting models on CEC2014 benchmark functions.(BP: benchmark problem, M: model)

BP	Statistic	M-1	M-2	M-3	M-4	M-5	M-6
F1	BEST	2.81E+06	1.40E+07	1.91E+07	1.65E+06	1.19E+09	2.97E+05
	MEAN	2.21E+07	8.75E+07	1.22E+08	7.88E+07	2.55E+09	2.30E+06
	STD	1.21E+07	4.98E+07	8.14E+07	9.46E+07	7.13E+08	3.30E+06
F2	BEST	2.53E+08	3.44E+09	4.77E+09	5.47E+08	6.88E+10	1.12E+03
	MEAN	1.18E+09	1.13E+10	1.51E+10	1.24E+10	1.09E+11	4.50E+07
	STD	7.88E+08	4.56E+09	7.60E+09	7.37E+09	1.46E+10	1.13E+08
F3	BEST	2508.161	7452.235	6718.339	1530.882	115288.26	12.31576
	MEAN	10283.878	23565.761	26412.623	17371.098	1339974.58	720.98662
	STD	7405.9652	11390.718	12987.686	11167.120	3977255.87	459.05083
F4	BEST	124.63317	237.20417	366.65430	99.931950	12469.940	1.7349235
	MEAN	261.72851	1248.1136	1524.8880	1117.3840	28719.912	101.73436
	STD	75.082818	970.04659	1026.3580	1051.5020	7180.2395	49.705396
F5	BEST	19.999919	19.999945	19.999984	20.001488	20.904741	19.99991
	MEAN	20.000471	20.027616	20.119284	20.203561	21.227639	20.051011
	STD	0.0017286	0.0676664	0.10996321	0.14716807	0.11656360	0.06805144
F6	BEST	12.780267	16.218868	14.020127	15.457791	43.923473	6.7032715
	MEAN	18.529760	22.064181	23.363359	24.965132	47.241693	15.522151
	STD	3.0297552	2.6811519	3.0934760	4.5806269	1.5137781	3.6245682
F7	BEST	4.1889379	36.384597	30.009411	20.649309	787.68143	0.0760718
	MEAN	10.809216	105.63397	145.81157	133.27976	1031.7814	1.5426381
	STD	4.5220568	46.492807	72.440511	84.883337	126.199427	1.3405380
F8	BEST	61.962747	72.316199	102.30215	103.36775	386.15642	32.833624
	MEAN	112.07644	138.87702	173.95711	180.68505	464.51985	63.563461
	STD	26.789164	30.133525	39.792817	38.237983	29.718694	16.456396
F9	BEST	96.328012	124.2611	133.30573	102.83446	452.21840	39.79830
	MEAN	156.90387	208.60209	199.09326	198.51539	542.47005	68.821782
	STD	33.954990	38.350229	32.418082	37.809694	38.086749	15.726623
F10	BEST	1210.0402	1626.4813	1723.7538	1163.3399	7654.5783	2135.4256
	MEAN	2351.0697	2890.8208	3254.8170	3891.3623	8777.0807	3526.7174
	STD	613.49072	727.70908	737.95259	839.32857	436.53375	591.53509
F11	BEST	2149.5498	2528.9014	2466.4730	2558.7366	7850.1910	2299.2129
	MEAN	3318.7230	3494.9934	3707.1753	4107.5731	8955.6194	3829.2650
	STD	552.91312	578.27788	603.92219	713.80344	487.54834	792.19294
F12	BEST	0.0332293	0.0216579	0.0450383	0.0147452	2.1111738	0.0061812
	MEAN	0.1013584	0.1118079	0.1240606	0.1311791	4.9438427	0.0886062
	STD	0.0595593	0.0600447	0.0724356	0.128588	1.0550659	0.1059418
F13	BEST	0.3398642	0.3358179	0.3325809	0.4149250	6.8122401	0.2706575
	MEAN	0.4932546	2.2437627	3.2495150	2.4684049	9.5480578	0.4745373
	STD	0.1034595	1.1874675	1.1134159	1.4082924	1.0074235	0.0984611
F14	BEST	0.1983121	12.646083	16.691299	0.2640280	219.72873	0.1357897
	MEAN	0.6238106	34.053169	56.271728	36.983065	352.13961	0.2521791
	STD	1.2595283	15.034313	24.679984	26.728169	52.830072	0.1142935
F15	BEST	7.9853189	163.90467	124.44512	14.207000	1698645.13	7.3577865
	MEAN	36.234616	16419.834	47313.228	31450.249	1.10E+07	16.043821
	STD	31.307650	22124.798	64889.212	61477.815	7.17E+06	4.6894993
F16	BEST	9.6608408	9.778447	10.0252178	9.0158682	12.855770	10.075036
	MEAN	11.265600	11.756873	11.708226	11.950595	14.011492	12.209291
	STD	0.7430629	0.7677885	0.7354599	0.8490922	0.2875473	0.7071105
F17	BEST	2.99E+04	1.21E+05	7.58E+04	4.52E+03	2.23E+07	482.45533
	MEAN	2.05E+05	1.41E+06	2.12E+06	2.67E+06	2.26E+08	1486.0755
	STD	1.45E+05	1.27E+06	3.53E+06	4.63E+06	1.17E+08	418.25419
F18	BEST	78.667759	255.22470	9259.0590	139.11947	1.25E+09	18.609534
	MEAN	4.91E+03	8.73E+07	3.55E+08	7.24E+07	6.61E+09	217.54539
	STD	5.50E+03	2.12E+08	4.19E+08	1.46E+08	2.11E+09	132.89962
F19	BEST	6.8626760	13.183991	15.962917	7.2304475	505.98221	2.8282229
	MEAN	19.077663	73.076117	96.777237	67.836319	893.46864	4.7783319
	STD	18.026375	33.681989	64.101520	50.403641	240.21494	1.1185220
F20	BEST	448.88476	1391.5927	647.28860	119.48715	5.94E+04	26.603055
	MEAN	7343.4016	1.56E+04	1.70E+04	9115.47485	8.76E+06	77.38887
	STD	5911.6221	1.33E+04	1.85E+04	1.06E+04	9.40E+06	33.064144
F21	BEST	3.83E+03	3.99E+03	2.36E+04	1.60E+03	2.66E+06	189.53629
	MEAN	4.88E+04	3.24E+05	5.77E+05	3.36E+05	9.61E+07	825.70941
	STD	4.30E+04	4.56E+05	7.69E+05	5.71E+05	6.40E+07	305.37384

Table 4 continue ...

BP	Statistic	M-1	M-2	M-3	M-4	M-5	M-6
F22	BEST	23.155044	300.07245	253.81527	147.17279	1790.8706	27.665821
	MEAN	466.81841	689.41870	713.42973	676.44776	6936.8034	405.85896
	STD	182.26092	211.84945	273.20237	266.19192	6822.7243	202.28829
F23	BEST	317.68609	327.11306	329.11530	319.78385	861.96608	315.25029
	MEAN	324.21290	380.67825	396.37542	355.22081	1565.4769	315.70547
	STD	4.5258656	61.022926	49.854870	27.685458	359.72836	0.5925139
F24	BEST	228.01370	259.44703	261.50067	248.53790	390.86832	222.78275
	MEAN	248.13671	285.38263	292.48831	283.94566	484.12895	236.98477
	STD	5.9780003	14.317518	18.130019	22.433450	35.988336	8.4137491
F25	BEST	204.13366	207.80684	207.57430	204.84942	271.15413	203.67564
	MEAN	211.38032	216.63161	217.96005	214.41357	363.27301	208.01792
	STD	2.7354476	4.0723565	5.1468239	13.030525	35.720118	2.9820432
F26	BEST	100.22110	100.33796	100.36468	100.24366	109.54589	100.27377
	MEAN	100.49711	101.97763	102.76053	101.68462	275.62835	100.47324
	STD	0.1321943	1.1105830	1.2366249	1.4070270	106.61013	0.1210883
F27	BEST	406.61930	409.65151	410.91979	402.62939	994.15148	400.88395
	MEAN	417.96541	477.56027	496.95538	593.58449	1789.9545	434.09199
	STD	9.5642998	96.532390	124.84061	220.40487	214.63327	86.993159
F28	BEST	1.077E+03	1.223E+03	1.309E+03	1.574E+03	6.542E+03	1.008E+03
	MEAN	1.689E+03	2.294E+03	2.167E+03	2.618E+03	8.170E+03	1.491E+03
	STD	543.27094	606.88783	670.03850	569.54248	826.83480	364.57574
F29	BEST	808.07603	6.041E+03	3.367E+03	1.305E+03	2.010E+08	475.78289
	MEAN	1.921E+03	2.271E+06	5.088E+06	2.544E+06	6.590E+08	1.052E+03
	STD	1.803E+03	3.162E+06	6.719E+06	4.501E+06	2.129E+08	290.47987
F30	BEST	2.018E+03	4.176E+03	4.952E+03	1.872E+03	7.959E+05	614.17566
	MEAN	6.898E+03	7.334E+04	9.143E+04	4.345E+04	6.469E+06	1.428E+03
	STD	4.322E+03	9.399E+04	9.280E+04	3.710E+04	3.212E+06	855.71695

Table 5: Average ranking of non-linear least square fitting models for CEC2014 benchmark functions.

Statistic	M1	M2	M3	M4	M5	M6
BEST	2.3	4.06	4.33	2.9	6	1.4
MEAN	1.83	3.4	4.46	3.86	6	1.43
STD	1.96	3.46	4.3	4.56	5.03	1.66

Table 6: Results of ILPE and LPE for CEC2014 benchmark functions

Function	Statistic	ILPE	LPE	Function	Statistic	ILPE	LPE
F1	BEST	2.97E+05	2.10E+07	F2	BEST	1.12E+03	7.22E+09
	MEAN	2.30E+06	7.77E+07		MEAN	4.50E+07	1.55E+10
	STD	3.30E+06	3.99E+07		STD	1.13E+08	4.57E+09
F3	BEST	12.315768	8.49E+03	F4	BEST	1.7349235	683.49216
	MEAN	720.98662	1.81E+04		MEAN	101.73436	1367.4426
	STD	459.05083	5.04E+03		STD	49.705396	497.93208
F5	BEST	19.999910	20.770302	F6	BEST	6.7032715	16.38785
	MEAN	20.051012	20.984057		MEAN	15.522151	21.39877
	STD	0.0680514	2.4942514		STD	3.6245682	2.49425
F7	BEST	0.0760718	73.30494	F8	BEST	32.833624	98.873270
	MEAN	1.5426381	146.76028		MEAN	63.563461	146.43280
	STD	1.3405380	42.789080		STD	16.456396	19.082338
F9	BEST	39.798330	101.10976	F10	BEST	2135.4256	4235.7648
	MEAN	68.821782	149.32156		MEAN	3526.7175	6545.5747
	STD	15.726623	22.239564		STD	591.53509	794.24460
F11	BEST	2299.2130	5676.674	F12	BEST	0.0061812	0.7881328
	MEAN	3829.2651	7055.1938		MEAN	0.0886062	2.1908967
	STD	792.19294	532.3634		STD	0.1059418	0.6719325
F13	BEST	0.2706575	0.52536	F14	BEST	0.1357897	19.809169
	MEAN	0.4745373	3.3260419		MEAN	0.2521791	60.3380
	STD	0.0984611	0.60044		STD	0.1142935	18.71688
F15	BEST	7.3577865	104.671603	F16	BEST	10.075036	12.00349
	MEAN	16.043821	3325.8469		MEAN	12.209290	12.817754
	STD	4.6894993	5504.1062		STD	0.7071105	0.30373
F17	BEST	482.45533	915.05636	F18	BEST	18.609534	55.54811
	MEAN	1486.0754	49904.3271		MEAN	217.54539	221.06017
	STD	418.25418	102718.8388		STD	132.89962	297.86028
F19	BEST	2.8282230	10.908138	F20	BEST	26.603055	49.614692
	MEAN	4.7783319	36.016722		MEAN	77.388878	1272.9582
	STD	1.1185221	21.94409		STD	33.064144	1894.8155
F21	BEST	189.53630	453.06573	F22	BEST	27.665821	30.346251
	MEAN	825.70942	2701.1862		MEAN	405.85897	323.6588
	STD	305.37384	4178.8423		STD	202.28829	200.57489
F23	BEST	315.25029	332.7117	F24	BEST	222.78275	248.31840
	MEAN	315.70547	315.2441		MEAN	236.98477	261.55212
	STD	0.5925140	19.025293		STD	8.4137490	6.5626319
F25	BEST	203.67564	210.74768	F26	BEST	100.27377	100.26612
	MEAN	208.01792	215.86386		MEAN	100.47324	101.6506
	STD	2.9820432	2.7120482		STD	0.1210882	0.8632091
F27	BEST	400.88395	416.65378	F28	BEST	1008.0327	1274.2160
	MEAN	434.09199	754.27934		MEAN	1490.6089	1652.1034
	STD	86.993159	176.37627		STD	364.57574	198.88130
F29	BEST	475.78289	957.73727	F30	BEST	614.17566	2072.9303
	MEAN	1052.2308	31901.279		MEAN	1427.8911	10075.4506
	STD	290.47987	132655.332		STD	855.71695	8384.0334

Table 7: Results of ILPE and LPE on CEC2017 benchmark functions

Function	Statistic	ILPE	LPE	Function	Statistic	ILPE	LPE
F1	BEST	818.21775	6.26E+09	F2	BEST	9.87E+10	2.95E+21
	MEAN	3.86E+06	1.18E+10		MEAN	2.99E+19	1.02E+30
	STD	9.65E+06	2.30E+03		STD	1.82E+20	5.49E+30
F3	BEST	1.93E+03	2.53E+04	F4	BEST	2.0318959	483.707431
	MEAN	1.27E+04	4.62E+04		MEAN	87.311911	1669.27809
	STD	7.01E+03	1.41E+04		STD	33.296865	922.15074
F5	BEST	43.778202	100.39077	F6	BEST	7.2399405	16.210609
	MEAN	78.611869	168.38508		MEAN	20.431315	23.558782
	STD	19.649532	27.186857		STD	6.6659069	3.8358785
F7	BEST	98.093454	214.36562	F8	BEST	29.848759	115.15877
	MEAN	164.70135	305.84287		MEAN	62.243968	154.4970
	STD	36.860912	51.661128		STD	16.825440	20.172154
F9	BEST	199.70024	722.88323	F10	BEST	1940.1120	6075.3759
	MEAN	971.14533	1772.4808		MEAN	3766.9330	7304.0427
	STD	361.83640	644.76856		STD	669.50612	441.2940
F11	BEST	21.043006	143.57571	F12	BEST	1.60E+03	1.96E+07
	MEAN	60.359185	337.10940		MEAN	2.61E+04	3.45E+08
	STD	21.896988	126.52227		STD	2.33E+04	3.85E+08
F13	BEST	243.20130	2.58E+03	F14	BEST	32.175573	34.412913
	MEAN	1.13E+03	1.52E+04		MEAN	67.993257	57.675041
	STD	742.29195	1.13E+04		STD	18.479353	14.204150
F15	BEST	44.211649	135.68332	F16	BEST	365.74394	128.86657
	MEAN	216.01333	784.43639		MEAN	970.42645	1055.2653
	STD	120.34630	628.72775		STD	335.73508	509.36832
F17	BEST	52.302877	55.311557	F18	BEST	49.807449	671.07190
	MEAN	375.53465	161.85635		MEAN	123.38171	12169.422
	STD	211.24538	131.82655		STD	53.427191	11700.914
F19	BEST	25.790923	21.492625	F20	BEST	162.6415	62.383571
	MEAN	93.666972	517.22217		MEAN	515.84561	352.59747
	STD	45.108947	1004.2185		STD	190.93070	226.03619
F21	BEST	224.82685	295.09652	F22	BEST	100.03665	599.99867
	MEAN	265.37033	344.89042		MEAN	106.15564	1236.3515
	STD	16.498367	25.337544		STD	7.0228152	358.94438
F23	BEST	416.78554	460.28107	F24	BEST	481.66111	587.17193
	MEAN	484.35757	555.31237		MEAN	553.30175	655.11318
	STD	40.500158	35.410313		STD	45.735658	38.036189
F25	BEST	384.20874	574.73761	F26	BEST	228.25327	1941.17404
	MEAN	404.53082	840.12162		MEAN	1880.1754	3420.8834
	STD	17.424591	134.09036		STD	827.71805	516.08168
F27	BEST	513.28631	524.23254	F28	BEST	380.70999	627.77775
	MEAN	565.89051	596.72511		MEAN	441.42617	1065.98558
	STD	29.316709	31.888075		STD	36.582659	267.21641
F29	BEST	476.57379	625.10540	F30	BEST	2.89E+03	1.95E+04
	MEAN	760.15080	783.791191		MEAN	8.12E+03	1.16E+05
	STD	181.17147	141.22810		STD	4.50E+03	1.14E+05

Table 8: Results of ILPE with other metaheuristic algorithms on CEC2014 benchmark functions

Function	Statistic	ILPE	GPE	GPEae	GPEed	PSO	DE	ABC	GWO	GA	TLBO
F1	BEST	2.97E+05	2.68E+06	3.81E+06	6.74E+06	6.33E+06	4.03E+07	9.16E+08	1.29E+09	4.05E+08	4.05E+08
	MEAN	2.30E+06	1.67E+07	1.37E+07	3.18E+07	3.99E+07	7.67E+07	1.37E+09	2.92E+09	4.73E+08	4.05E+08
	STD	3.30E+06	1.03E+07	7.35E+06	1.72E+07	3.28E+07	1.93E+07	2.26E+08	8.46E+08	1.77E+08	8.80E-07
F2	BEST	1.12E+03	6.18E+07	1.96E+08	6.23E+08	1.13E+08	2.72E-05	5.45E+10	1.00E+11	5.25E+10	5.25E+10
	MEAN	4.50E+07	8.17E+08	9.30E+08	2.73E+09	8.50E+08	1.80E-04	6.86E+10	1.37E+11	5.25E+10	5.25E+10
	STD	1.13E+08	6.01E+08	6.97E+08	1.41E+09	6.57E+08	1.28E-04	7.00E+09	2.75E+10	5.11E+03	4.213432
F3	BEST	12.31576	1.23E+03	1.20E+03	3.45E+03	394.5023	6.96E-03	7.30E+04	2.14E+05	5.08E+04	5.05E+04
	MEAN	720.9866	9.67E+03	7.56E+03	1.32E+04	3.65E+03	2.10E-02	1.05E+05	3.95E+05	1.34E+05	5.06E+04
	STD	459.0508	5.56E+03	4.40E+03	5.37E+03	3.53E+03	1.10E-02	1.72E+04	6.90E+04	5.74E+04	410.9774
F4	BEST	1.734923	132.1708	123.0414	144.4509	107.2276	73.5498	6434.050	14611.33	6039.589	6039.588
	MEAN	101.7343	241.0425	236.2078	365.9799	231.3592	81.38307	8857.150	3.39E+04	6294.866	6088.972
	STD	49.70539	66.83605	71.71590	128.8327	84.78022	11.70648	1181.631	1.39E+04	756.5473	22.10547
F5	BEST	19.99991	20.00000	20.84567	20.75067	20.41480	20.77374	20.45456	20.41403	20	20.80694
	MEAN	20.05101	20.38529	21.01877	21.03882	20.68511	20.95197	20.56069	20.80769	20.10450	20.96744
	STD	0.068051	0.342099	0.054897	0.07030	0.126151	0.056718	0.041883	0.118807	0.1009665	0.052340
F6	BEST	6.703271	18.05249	16.47943	17.87789	30.15295	1.788182	24.12283	20.51959	15.66169	13.22175
	MEAN	15.52215	24.67914	22.84872	23.01308	36.27924	25.42390	27.00712	28.02580	25.07317	19.88990
	STD	3.624568	3.055995	3.140175	2.692611	2.803966	5.762718	1.221580	3.275361	3.229920	3.106897
F7	BEST	0.07607	1.946567	2.743728	5.100878	1.587741	2.27E-13	860.1497	1429.075	713.3710	713.3710
	MEAN	1.542638	9.049820	9.243718	20.25779	10.89152	1.45E-04	1030.520	2160.848	713.3710	713.3710
	STD	1.340538	4.694226	4.619875	10.08948	1.17E+01	1.04E-03	8.93E+01	3.47E+02	1.64E-04	2.67E-13
F8	BEST	32.83362	37.62503	43.09574	44.50626	91.25442	92.84614	215.1083	319.4705	122.1789	160.7306
	MEAN	63.56346	79.62665	75.06652	84.01618	130.4014	111.6110	276.6328	506.9172	163.1726	254.7089
	STD	16.45639	22.16072	17.03879	19.61313	18.76299	8.266794	26.68490	70.22687	27.52555	43.24035
F9	BEST	39.79833	45.01157	47.91852	63.47771	116.3077	165.8749	331.5778	404.9647	176.9715	141.2645
	MEAN	68.82178	93.18807	96.39749	92.82069	166.4212	186.2279	495.3122	648.4874	251.4603	225.6659
	STD	15.72662	24.27488	27.15970	18.37806	26.24150	9.86628	49.06103	126.2525	43.21568	39.63705
F10	BEST	2135.425	1577.827	1391.446	2492.265	2062.462	3309.942	3672.628	4903.166	2436.611	2733.482
	MEAN	3526.717	3922.643	3546.398	5220.595	3902.101	4016.184	4198.662	6106.936	3509.719	4390.149
	STD	591.5350	1496.462	1528.239	1937.956	862.6573	286.2155	223.5060	560.7998	595.6909	1002.485
F11	BEST	2299.213	3222.658	3292.446	4504.100	3688.088	6068.423	4454.537	3228.333	3013.148	5017.700
	MEAN	3829.265	5065.500	7375.761	7387.861	5597.490	6805.900	5014.005	5092.728	4235.846	6750.903
	STD	792.1929	1421.488	856.7280	794.2484	1000.826	311.8117	212.5242	666.1863	657.9730	696.9054

Table 8 continue ...

Function	Statistic	ILPE	GPE	GPEae	GPEed	PSO	DE	ABC	GWO	GA	TLBO
F12	BEST	0.006181	0.056693	0.247131	1.383817	0.67319	1.81888	0.519413	0.46970	0.231104	1.674398
	MEAN	0.088606	0.692675	2.857136	2.987856	1.865055	2.317402	0.9180534	0.987723	0.695191	2.410485
	STD	0.105941	1.051771	0.570334	0.470912	0.542900	0.241179	0.118286	0.444115	0.298416	0.3143640
F13	BEST	0.270657	0.332569	0.325171	0.187935	0.316754	0.281139	6.407087	8.534468	6.153908	6.153908
	MEAN	0.474537	0.546358	0.534307	0.517344	0.550548	0.423477	7.241119	10.99172	6.153949	6.153908
	STD	0.098461	0.119760	0.112765	0.119351	0.123011	0.050002	0.386006	1.312680	1.15E-04	8.48E-13
F14	BEST	0.135789	0.158862	0.123495	0.218481	0.213814	0.220947	364.1341	611.6576	346.8616	346.8616
	MEAN	0.252179	0.504428	0.432839	6.174487	2.113097	0.285559	4.44E+02	8.32E+02	3.47E+02	3.47E+02
	STD	0.114293	1.327939	1.145509	6.962390	3.889626	2.83E-02	37.17637	109.8580	8.66E-04	2.74E-13
F15	BEST	7.357786	15.61953	15.32501	23.90051	120.9789	14.73525	7.37E+05	1.44E+07	4.45E+05	4.45E+05
	MEAN	16.04382	38.40215	37.78489	63.52328	707.1546	17.34197	2.24E+06	1.22E+08	4.45E+05	4.45E+05
	STD	4.689499	16.06414	13.27885	37.55980	706.253	0.895731	1.14E+06	7.63E+07	1.18E+03	28.22941
F16	BEST	10.07503	10.72562	10.88051	11.70663	11.26784	11.82336	11.98357	11.65554	11.73377	10.86452
	MEAN	12.20929	12.37030	12.69197	12.99174	12.35957	12.68299	12.52382	12.78761	12.77558	12.32245
	STD	0.707110	0.620654	0.641222	0.358621	0.492973	0.214624	0.200475	0.483282	0.483705	0.4492451
F17	BEST	482.4553	6.62E+03	5.73E+03	3.51E+04	4.42E+04	6.94E+05	1.51E+08	8.50E+07	6.98E+07	6.98E+07
	MEAN	1.49E+03	1.25E+05	1.03E+05	2.60E+05	7.70E+05	2.09E+06	2.47E+08	2.07E+08	7.10E+07	6.98E+07
	STD	418.2541	1.43E+05	1.15E+05	2.27E+05	8.36E+05	7.12E+05	4.19E+07	7.07E+07	5.71E+06	407.3107
F18	BEST	18.60953	84.16155	105.2904	121.0840	402.1450	1.00E+03	4.58E+09	4.56E+09	4.56E+09	4.56E+09
	MEAN	217.5453	2945.317	2.97E+03	2.54E+03	5.34E+05	5.65E+03	4.71E+09	5.93E+09	4.56E+09	4.56E+09
	STD	132.8996	2734.668	4.38E+03	2.40E+03	1.04E+06	4.44E+03	1.10E+08	2.04E+09	204.7946	136.6478
F19	BEST	2.828222	6.618417	6.861392	7.960605	28.11828	5.487725	488.7395	570.5980	456.2768	453.7356
	MEAN	4.778331	25.67592	29.88258	27.78672	122.2860	6.816235	602.6920	956.2438	471.4896	457.8511
	STD	1.118522	25.12992	28.14716	24.99503	79.94213	0.899387	73.20375	424.2245	51.80302	2.080066
F20	BEST	26.60305	144.8748	216.8829	143.2965	356.9527	197.1259	1.52E+05	2.04E+05	9.98E+04	7.86E+04
	MEAN	77.38887	3252.521	1315.863	3026.290	3735.841	310.5768	2.23E+05	3.76E+06	1.98E+05	7.96E+04
	STD	33.06414	3803.584	3048.077	4469.993	2762.860	82.78483	4.30E+04	4.84E+06	5.42E+04	1.76E+03
F21	BEST	189.5362	3029.417	2484.889	2123.288	5773.462	4.97E+04	1.27E+08	1.28E+08	1.20E+08	1.20E+08
	MEAN	825.7094	2.99E+04	1.99E+04	2.96E+04	1.68E+05	1.75E+05	1.71E+08	1.62E+08	1.20E+08	1.20E+08
	STD	305.3738	3.68E+04	1.87E+04	2.64E+04	2.07E+05	8.15E+04	2.59E+07	4.86E+07	3.19E+03	392.7588
F22	BEST	27.66582	209.1336	181.1109	188.9107	523.9290	101.0456	3.65E+03	3.22E+03	3.13E+03	2.95E+03
	MEAN	405.8589	687.9421	593.7321	568.2463	1.01E+03	312.0850	4.71E+03	6.53E+03	3.70E+03	3.26E+03
	STD	202.2882	214.8584	235.7795	225.5080	285.6837	81.75702	498.9252	4645.189	301.6119	190.5229

Table 8 Continue ...

Function	Statistic	ILPE	GPE	GPEae	GPEed	PSO	DE	ABC	GWO	GA	TLBO
F23	BEST	315.2502	317.2622	318.0933	319.8797	319.2362	315.2441	589.1315	825.2907	550.3770	550.3758
	MEAN	315.7054	321.2342	322.1293	328.2128	344.0166	315.2441	664.4085	1155.298	564.8492	550.3758
	STD	0.592513	2.786113	2.539552	4.994074	19.54930	7.73E-09	39.93381	260.7947	30.23120	3.06E-12
F24	BEST	222.7827	228.4938	240.0251	228.7121	232.0050	204.9038	286.0874	371.9970	200.0632	226.5933
	MEAN	236.9847	245.8164	248.4887	243.2869	242.7488	222.8829	308.5267	495.7983	238.5308	265.4885
	STD	8.413749	7.650002	4.873837	6.518096	8.079192	2.722102	12.71744	69.52281	25.64975	8.778038
F25	BEST	203.6756	207.0308	210.6106	208.0685	214.227	217.1990	216.6095	225.6092	205.3090	203.8945
	MEAN	208.0179	215.0370	215.9991	215.5135	224.2165	222.6526	226.8138	272.0063	211.7596	205.5559
	STD	2.982043	5.476376	3.673484	3.803373	6.831476	2.966085	4.866395	37.95738	6.974553	1.088413
F26	BEST	100.2737	100.2893	100.3065	100.2815	100.30896	100.2870	109.0519	267.0357	108.8513	108.8513
	MEAN	100.4732	100.5026	100.5215	102.4613	173.4399	100.4131	109.7263	779.4293	357.5181	165.1220
	STD	0.121088	0.125551	0.158361	13.97349	44.70510	0.052170	0.419885	444.4853	396.9623	69.63745
F27	BEST	400.8839	416.7631	411.9468	408.6019	413.4737	300.0642	981.3728	1034.244	838.1432	793.7189
	MEAN	434.0919	933.6711	840.1615	826.9095	1248.548	300.3253	1204.074	1193.891	1069.613	939.4042
	STD	86.99315	171.9440	160.0676	208.1399	599.2142	0.314638	69.94510	80.45672	104.4848	77.44285
F28	BEST	1008.032	1296.195	1013.763	1172.666	3856.301	972.4042	741.1137	868.2711	482.9409	490.1160
	MEAN	1490.608	1666.860	1498.355	1756.757	5305.735	1040.032	898.5941	1355.346	577.6819	597.3079
	STD	364.5757	199.8184	336.4432	313.3783	848.0666	32.46447	76.80525	361.6102	71.94769	75.85007
F29	BEST	475.7828	1264.059	1172.460	1160.917	3.98E+03	6.41E+03	6.36E+05	1.91E+06	5.11E+05	4.87E+05
	MEAN	1052.230	2660.003	1920.472	2770.533	5.01E+06	1.85E+04	8.09E+05	7.46E+06	5.36E+05	5.61E+05
	STD	290.4798	1082.704	807.2368	1621.790	7.96E+06	6.67E+03	8.66E+04	4.81E+06	2.48E+04	5.91E+04
F30	BEST	614.1756	3672.738	2.61E+03	2.83E+03	5.95E+03	3.66E+03	9.83E+03	4.84E+04	5.59E+03	5.74E+03
	MEAN	1.43E+03	2.02E+04	8.70E+03	1.50E+04	6.03E+04	5.40E+03	1.41E+04	1.68E+05	8.29E+03	9.08E+03
	STD	855.7169	2.08E+04	5.60E+03	1.02E+04	1.26E+05	978.90334	2.36E+03	9.12E+04	2.40E+03	3.35E+03

Table 9: Results of ILPE with other metaheuristic algorithms on CEC2017 benchmark functions

Function	Statistic	ILPE	GPE	GPEae	GPEed	PSO	DE	ABC	GWO	GA	TLBO
F1	BEST	818.217	1.45E+07	3.39E+11	4.64E+08	1.02E+08	158.685	3.59E+10	6.53E+10	3.10E+10	3.10E+10
	MEAN	3.86E+06	5.03E+08	3.59E+11	1.54E+09	7.52E+08	2.52E+03	4.47E+10	1.09E+11	3.10E+10	3.10E+10
	STD	9.65E+06	3.44E+08	7.24E+09	9.14E+08	7.04E+08	1.60E+03	3.16E+09	5.42E+09	3.03837	2.60E-05
F2	BEST	9.87E+10	4.27E+16	4.69E+91	6.13E+18	1.21E+18	9.66E+24	1.66E+34	9.33E+34	1.45E+34	1.45E+34
	MEAN	2.99E+19	2.66E+25	3.95E+93	6.58E+26	3.71E+31	3.53E+27	3.86E+34	2.50E+44	4.14E+37	1.45E+34
	STD	1.82E+20	1.89E+26	5.31E+93	4.06E+27	1.36E+32	8.43E+27	4.45E+34	1.72E+45	2.96E+38	1.00E+25
F3	BEST	1.93E+03	8.51E+03	4.76E+05	1.08E+04	227.462	5.49E+04	2.68E+05	2.19E+05	9.73E+04	3.67E+04
	MEAN	1.27E+04	3.17E+04	5.26E+05	3.91E+04	990.565	6.84E+04	3.47E+05	3.62E+05	2.21E+05	3.69E+04
	STD	7.01E+03	1.10E+04	2.83E+04	1.05E+04	729.125	8.42E+03	2.77E+04	6.39E+04	7.55E+04	346.420
F4	BEST	2.03189	95.0121	2.10E+05	183.980	108.321	85.4857	2.64E+04	3.80E+04	2.51E+04	2.51E+04
	MEAN	87.3119	214.786	2.44E+05	348.879	224.387	98.0296	3.20E+04	8.42E+04	2.52E+04	2.52E+04
	STD	33.2968	67.8574	1.31E+04	97.1974	183.501	14.2266	3.07E+03	2.78E+04	69.7965	69.8550
F5	BEST	43.7782	65.8838	1.44E+03	66.8365	157.627	153.598	440.335	488.794	226.031	210.26348
	MEAN	78.6118	114.279	1.55E+03	120.043	212.673	187.203	564.610	718.408	303.377	289.576
	STD	19.6495	26.9440	36.7095	26.3224	29.9236	9.29166	42.3298	117.997	53.1798	42.1063
F6	BEST	7.23994	17.4616	139.531	14.7385	41.1641	1.23E-07	97.7044	126.392	52.65481	48.09306
	MEAN	20.4313	27.8279	152.196	27.8260	55.3638	7.40E-07	117.972	156.454	77.9135	57.8178
	STD	6.66590	5.42308	4.80237	6.98841	5.53728	6.16E-07	7.09435	15.4705	17.1255	7.50588
F7	BEST	98.0934	165.264	6.96E+03	178.540	254.321	200.850	695.439	1.55E+03	414.0013	379.078
	MEAN	164.701	293.989	7.49E+03	287.148	403.302	222.845	1.01E+03	2.69E+03	496.363	438.546
	STD	36.8609	62.12038	176.603	66.53639	86.6981	10.6360	176.695	602.221	50.2757	36.7282
F8	BEST	29.8487	49.01994	1.76E+03	54.82313	125.5605	158.7368	363.4164	292.436	154.4267	143.8903
	MEAN	62.2439	78.59719	1.89E+03	87.84680	163.6584	187.0717	445.4893	515.6562	231.4999	233.4174
	STD	16.82544	19.26963	42.43474	14.97565	24.09724	9.454429	35.48068	78.10719	67.60613	51.30958
F9	BEST	199.7002	489.2612	8.51E+04	617.1419	1.93E+03	1.14E-13	1.87E+04	2.29E+04	5.23E+03	3.04E+03
	MEAN	971.1453	1.45E+03	9.91E+04	1.59E+03	3.75E+03	5.08E-13	2.84E+04	3.72E+04	2.01E+04	5.55E+03
	STD	361.8363	609.9913	5.59E+03	434.6180	995.5729	6.60E-13	3.28E+03	1.16E+04	7.76E+03	1.37E+03
F10	BEST	1940.112	2950.209	1.44E+04	4209.697	3622.546	6152.888	4343.652	3509.765	2693.999	4686.376
	MEAN	3766.932	3989.589	1.55E+04	7325.763	6236.466	7054.500	4980.011	4817.360	4323.309	7279.103
	STD	669.5061	546.0398	422.3134	1001.867	1210.354	272.7909	276.00259	622.1292	777.2029	652.5143
F11	BEST	21.04300	60.44106	7.59E+04	71.9571	145.1092	73.7136	4452.387	4231.492	2869.561	2842.296
	MEAN	60.35918	152.9092	8.79E+04	202.9353	334.4750	116.1398	7935.867	2.05E+04	3428.506	2992.573
	STD	21.89698	41.57286	6014.604	75.48644	123.7378	22.49259	1855.244	9985.501	1078.462	103.9775

Table 9 continue ...

Function	Statistic	ILPE	GPE	GPEae	GPEed	PSO	DE	ABC	GWO	GA	TLBO
F12	BEST	1.60E+03	4.65E+04	2.07E+11	3.67E+05	7.23E+06	9.98E+05	1.50E+10	1.88E+10	1.48E+10	1.48E+10
	MEAN	2.61E+04	4.08E+06	2.22E+11	1.06E+07	7.61E+07	4.68E+06	1.62E+10	2.67E+10	1.48E+10	1.48E+10
	STD	2.33E+04	2.75E+06	6.80E+09	1.24E+07	1.10E+08	2.11E+06	8.87E+08	8.27E+09	7.19E+06	360.8782
F13	BEST	243.2012	840.1494	1.29E+11	1.31E+03	1.37E+04	6.94E+03	3.03E+10	3.03E+10	3.03E+10	3.03E+10
	MEAN	1.13E+03	2.36E+04	1.43E+11	2.16E+04	1.38E+07	4.40E+04	3.16E+10	3.65E+10	3.03E+10	3.03E+10
	STD	742.2919	2.01E+04	5.43E+09	1.80E+04	6.90E+07	2.61E+04	1.17E+09	7.52E+09	6.74E+03	1300.189
F14	BEST	32.17557	57.54884	5.99E+08	55.30407	107.6116	166.1781	5.60E+06	5.35E+06	5.31E+06	5.31E+06
	MEAN	67.99325	128.3837	1.07E+09	298.4104	5280.871	274.0569	8.90E+06	9.74E+06	5.78E+06	5.31E+06
	STD	18.47935	772.5407	1.54E+08	613.5483	1.15E+04	7.43E+01	2.19E+06	1.01E+07	5.48E+05	135.7497
F15	BEST	44.21164	251.9588	6.25E+10	185.7015	1454.046	615.1236	6.18E+09	6.18E+09	6.18E+09	6.18E+09
	MEAN	216.0133	5578.218	7.18E+10	4608.141	1.37E+04	1130.283	6.21E+09	6.18E+09	6.18E+09	6.18E+09
	STD	120.3463	5030.645	3.67E+09	5318.920	9874.292	390.4139	3.41E+07	1.04E+05	2.67E+04	319.0359
F16	BEST	365.7439	495.2369	3.20E+04	378.8452	925.0950	1052.788	1532.666	1300.916	827.686	615.9068
	MEAN	970.4264	1083.691	3.61E+04	1070.162	1856.913	1353.540	2133.968	2551.263	1695.508	1130.528
	STD	335.7350	354.1858	2.00E+03	287.6113	569.7131	136.8576	190.9774	509.8836	385.4888	265.3326
F17	BEST	52.30287	177.2380	7.34E+06	95.07987	230.6451	154.2892	5.07E+03	5.24E+03	4.51E+03	4.58E+03
	MEAN	375.5346	625.6277	1.56E+07	449.3965	868.4829	265.9126	6.30E+03	5.94E+03	5.24E+03	5.08E+03
	STD	211.2453	284.1323	3.83E+06	218.5697	327.3078	58.69909	569.3697	372.2972	316.4307	268.8284
F18	BEST	49.80744	1010.750	1.34E+09	1505.766	8468.979	2.43E+05	6.05E+05	3.32E+05	2.49E+05	1.69E+05
	MEAN	123.3817	2.48E+04	2.07E+09	3.17E+04	8.77E+04	1.14E+06	2.37E+06	6.29E+06	2.22E+06	4.70E+05
	STD	53.42719	2.64E+04	3.16E+08	2.28E+04	1.10E+05	4.27E+05	1.08E+06	8.67E+06	2.04E+06	2.30E+05
F19	BEST	25.79092	57.26424	1.15E+10	64.84361	273.8911	138.9001	6.65E+09	6.65E+09	6.65E+09	6.65E+09
	MEAN	93.66697	6597.417	1.33E+10	6069.809	1.51E+05	566.0942	6.65E+09	6.68E+09	6.65E+09	6.65E+09
	STD	45.10894	9020.366	8.50E+08	6699.183	4.25E+05	338.1140	9.94E+05	8.83E+07	230.8695	160.5709
F20	BEST	162.6415	264.4236	3004.969	112.4908	319.1705	160.6657	875.6783	843.2133	635.1233	558.6650
	MEAN	515.8456	629.5345	3488.388	614.6152	729.6200	332.1061	1059.415	1081.530	1070.359	808.1065
	STD	190.9307	204.7104	201.1066	271.2205	229.2918	81.64216	95.3650	131.9351	172.0493	134.7046
F21	BEST	224.8268	256.9077	1572.629	258.3619	368.9531	360.2168	683.2313	678.5329	463.2610	449.0994
	MEAN	265.3703	301.8511	1641.236	310.4279	449.9923	381.8061	796.6096	857.7341	559.0712	521.3453
	STD	16.49836	23.26631	31.65160	23.54876	45.77366	9.930076	39.28954	82.90791	56.77391	41.39507
F22	BEST	100.0366	214.1088	1.40E+04	210.5056	698.8834	100	4711.591	3947.814	3406.914	4321.393
	MEAN	106.1556	3900.133	1.54E+04	2963.137	5220.406	100	5236.438	4986.030	4490.681	7253.658
	STD	7.022815	1823.122	378.4033	3459.629	1513.409	1.90E-09	229.2462	653.3566	580.8951	660.9192

Table 9 continue ...

Function	Statistic	ILPE	GPE	GPeae	GPEed	PSO	DE	ABC	GWO	GA	TLBO
F23	BEST	416.7855	396.3128	4767.669	447.3977	920.3256	499.0932	877.4908	820.2636	680.0859	627.9119
	MEAN	484.3575	491.7578	5176.264	510.5054	1201.360	533.2032	948.3525	970.6046	787.6332	718.4222
	STD	40.50015	33.39570	142.3933	44.61731	135.2205	11.71488	36.54075	100.5257	55.95695	44.03807
F24	BEST	481.6611	484.3333	4448.218	501.4884	758.9926	583.2577	1131.985	899.4952	794.5638	746.0107
	MEAN	553.3017	564.1546	4796.835	600.4007	1106.258	607.2859	1204.069	1060.999	908.5531	790.0989
	STD	45.73565	36.10579	127.4977	48.26721	179.5462	11.18463	39.58459	91.11655	55.02330	29.6622
F25	BEST	384.2087	421.6945	5.94E+04	441.1804	421.6659	386.7459	2772.160	8820.029	2360.004	2360.001
	MEAN	404.5308	474.0160	6.71E+04	521.7355	523.2680	386.9786	3842.459	1.98E+04	2410.787	2424.973
	STD	17.42459	32.34708	3461.474	49.26890	56.74966	0.131063	555.0659	6509.611	93.15141	50.3801
F26	BEST	228.2532	1969.328	8.18E+04	2093.614	669.8747	2467.809	5606.197	5586.144	3938.297	3794.463
	MEAN	1880.175	2554.576	9.11E+04	2730.366	5997.324	2739.778	6923.939	7071.817	5224.107	4661.193
	STD	827.7180	355.2852	3856.567	373.7200	1565.005	125.7286	392.8329	768.6203	687.4317	507.4709
F27	BEST	513.2863	533.0333	2.39E+04	554.0061	1043.136	517.5310	735.3850	910.2496	687.8988	687.8983
	MEAN	565.8905	588.5632	2.57E+04	607.9241	1692.632	539.1910	810.6678	1237.323	693.3996	694.0722
	STD	29.31670	33.89701	953.3144	33.05769	325.2914	8.487746	31.90744	201.7662	7.473493	9.320571
F28	BEST	380.7099	444.6031	2.93E+04	473.2055	458.7614	395.2469	4493.484	5494.451	3388.431	3388.431
	MEAN	441.4261	542.7703	3.17E+04	632.9271	576.7912	417.1822	7634.395	1.39E+04	6846.205	6252.259
	STD	36.58265	59.65900	948.2485	95.31664	82.99650	14.50945	1338.289	4038.862	3862.794	3942.957
F29	BEST	476.5737	616.4484	5.96E+06	623.1528	1276.614	876.4021	1181.587	1133.165	911.7886	763.825
	MEAN	760.1508	1043.144	1.25E+07	960.7918	2258.057	1091.543	1570.845	1780.813	1432.914	1082.265
	STD	181.1714	190.1090	2.92E+06	149.5946	528.9956	119.4874	176.0880	435.6054	295.2931	170.0793
F30	BEST	2.89E+03	6.40E+03	1.90E+10	7973.801	5.29E+05	7.23E+04	1.64E+09	2.42E+09	1.53E+09	1.53E+09
	MEAN	8.12E+03	3.68E+04	2.18E+10	8.03E+04	8.43E+06	1.97E+05	1.91E+09	4.88E+09	1.57E+09	1.53E+09
	STD	4.50E+03	3.41E+04	1.18E+09	8.65E+04	8.99E+06	7.57E+04	1.63E+08	8.36E+08	9.97E+07	0.118322

Table 10: **Average ranking of ILPE and other considered algorithms for CEC2014 benchmark functions.**

Statistic	ILPE	GPE	GPEae	GPEed	LPE	PSO	DE	ABC	GWO	GA	TLBO
BEST	1.63	4.2	4.47	5.2	5.9	5.9	5.07	9.43	9.77	7.07	7.37
MEAN	1.6	4.43	4.83	6	6.1	6.37	3.93	8.63	9.97	7	7.13
STD	3.73	6.5	5.67	6.33	5.93	7.8	2.93	6.97	9.7	6.6	3.83

Table 11: **Average ranking of ILPE and other considered algorithms for CEC2017 benchmark functions.**

Statistic	ILPE	GPE	GPEae	GPEed	LPE	PSO	DE	ABC	GWO	GA	TLBO
BEST	1.37	3	11	3.63	4.1	5.6	4.47	9.33	9.17	7.5	6.83
MEAN	1.43	3.2	10.97	4	4.1	6.17	3.47	8.8	9.5	7.53	6.83
STD	3.27	4.8	9.2	5.5	5	7.57	2.17	7.17	9.6	7.17	4.57

Table 12: Wilcoxon signed-rank test for CEC2014 benchmark functions.(BP:benchmark problem, p- value=p-val)

BP		GPE	GPEae	GPEed	LPE	PSO	DE	ABC	GWO	GA	TLBO
F1	p-val	6.315E-17	9.649E-16	3.942E-18	3.304E-18	5.945E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.175E-18
	rank	+	+	+	+	+	+	+	+	+	+
F2	p-val	8.707E-15	3.783E-17	3.504E-18	3.304E-18	7.431E-15	3.304E-18	3.304E-18	3.304E-18	3.301E-18	3.304E-18
	rank	+	+	+	+	+	-	+	+	+	+
F3	p-val	2.842E-17	1.598E-17	3.304E-18	3.304E-18	2.205E-08	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	-	+	+	+	+
F4	p-val	3.723E-15	1.639E-13	4.181E-18	3.304E-18	2.749E-14	5.962E-04	3.304E-18	3.304E-18	3.304E-18	3.297E-18
	rank	+	+	+	+	+	+	+	+	+	+
F5	p-val	3.381E-14	3.304E-18	3.304E-18	3.304E-18	3.504E-18	3.304E-18	3.304E-18	3.304E-18	5.291E-05	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F6	p-val	1.076E-15	2.571E-13	5.370E-14	4.846E-14	3.304E-18	3.170E-15	3.504E-18	4.703E-18	6.227E-16	1.504E-10
	rank	+	+	+	+	+	-	+	+	+	+
F7	p-val	3.927E-15	6.933E-14	5.031E-17	3.304E-18	5.406E-15	3.266E-18	3.304E-18	3.304E-18	1.791E-18	2.313E-18
	rank	+	+	+	+	+	-	+	+	+	+
F8	p-val	4.738E-03	8.309E-05	3.034E-09	3.304E-18	4.181E-18	7.073E-17	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F9	p-val	7.400E-10	2.941E-12	3.381E-14	4.181E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F10	p-val	1	1	3.361E-08	3.304E-18	7.576E-03	9.257E-13	2.234E-14	3.304E-18	7.095E-06	1.955E-05
	rank	≈	≈	+	+	-	+	+	+	+	+
F11	p-val	2.229E-10	3.588E-16	1.343E-17	7.963E-18	2.571E-13	3.304E-18	1.472E-14	3.138E-13	3.296E-06	5.637E-17
	rank	+	+	+	+	+	+	+	+	+	+
F12	p-val	7.560E-12	3.942E-18	3.504E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	4.181E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F13	p-val	1.303E-02	4.310E-04	4.738E-03	3.304E-18	3.028E-03	2.385E-04	3.304E-18	3.304E-18	2.519E-18	3.052E-18
	rank	+	+	-	+	+	+	+	+	+	+
F14	p-val	2.139E-05	1.351E-06	2.106E-13	3.304E-18	4.079E-10	4.840E-10	3.304E-18	3.304E-18	1.932E-18	2.314E-18
	rank	+	-	+	+	+	+	+	+	+	+
F15	p-val	6.684E-15	1.954E-15	9.927E-17	3.304E-18	3.304E-18	9.784E-03	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F16	p-val	1	3.167E-07	7.922E-12	3.837E-03	4.911E-02	1.291E-04	2.369E-02	5.266E-04	3.089E-06	1
	rank	≈	+	+	+	+	+	+	+	+	≈

Table 12 continue ...

BP		GPE	GPEae	GPEed	LPE	PSO	DE	ABC	GWO	GA	TLBO
F17	p-val	3.504E-18	3.304E-18	3.304E-18	6.227E-16	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F18	p-val	2.549E-12	4.437E-13	5.184E-11	1	3.717E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	≈	+	+	+	+	+
F19	p-val	9.136E-16	1.065E-17	3.504E-18	4.476E-16	5.607E-18	1.440E-10	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F20	p-val	3.304E-18	1.268E-17	3.942E-18	9.178E-15	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F21	p-val	3.942E-18	3.942E-18	3.717E-18	2.473E-09	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F22	p-val	1.064E-05	9.015E-04	2.170E-02	9.686E-04	6.684E-15	1.009E-04	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F23	p-val	4.434E-18	4.987E-18	3.942E-18	3.304E-18	3.504E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	2.994E-18
	rank	+	+	+	+	+	-	+	+	+	+
F24	p-val	2.885E-08	1.000E-11	3.768E-08	3.304E-18	9.286E-05	7.085E-18	3.304E-18	3.304E-18	6.099E-05	3.946E-14
	rank	+	+	+	+	+	-	+	+	-	+
F25	p-val	1.792E-10	1.020E-12	1.075E-14	9.383E-17	3.573E-17	3.304E-18	1.343E-17	3.304E-18	1	1.154E-07
	rank	+	+	+	+	+	+	+	+	≈	+
F26	p-val	1.457E-02	2.580E-04	1.141E-02	3.399E-07	4.241E-17	1	3.304E-18	3.304E-18	3.302E-18	3.274E-18
	rank	+	+	+	+	+	≈	+	+	+	+
F27	p-val	3.009E-17	1.424E-17	2.176E-16	9.383E-17	2.106E-13	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	-	+	+	+	+
F28	p-val	1	1	2.711E-06	1	3.304E-18	1.574E-15	2.534E-17	2.263E-04	5.288E-18	3.304E-18
	rank	≈	≈	+	≈	+	-	-	-	-	-
F29	p-val	4.181E-18	2.006E-12	5.280E-16	1.814E-14	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.296E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F30	p-val	4.434E-18	3.304E-18	3.304E-18	1.794E-17	3.304E-18	3.717E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
total '+', symbol		27	27	29	27	29	21	29	29	27	28

Table 13: Wilcoxon signed-rank test for CEC2017 benchmark functions.(BP:benchmark problem, p-value =p-val)

BP		GPE	GPEae	GPEed	LPE	PSO	DE	ABC	GWO	GA	TLBO
F1	p-val	3.500E-18	3.304E-18	3.304E-18	3.300E-18	4.430E-18	9.110E-12	3.300E-18	3.300E-18	3.270E-18	2.960E-18
	rank	+	+	+	+	+	-	+	+	+	+
F2	p-val	8.260E-15	3.304E-18	1.076E-15	3.720E-18	4.730E-16	3.300E-18	3.300E-18	3.300E-18	3.300E-18	3.290E-18
	rank	+	+	+	+	+	+	+	+	+	+
F3	p-val	2.610E-14	3.304E-18	1.076E-15	4.703E-18	3.504E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	-	+	+	+	+	+
F4	p-val	1.598E-17	3.304E-18	1.508E-17	3.304E-18	1.391E-16	1	3.304E-18	3.304E-18	3.304E-18	3.266E-18
	rank	+	+	+	+	+	≈	+	+	+	+
F5	p-val	1.300E-12	3.304E-18	3.731E-12	3.304E-18						
	rank	+	+	+	+	+	+	+	+	+	+
F6	p-val	4.665E-07	3.304E-18	5.749E-07	1.411E-03	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	-	+	+	+	+
F7	p-val	1.050E-16	3.304E-18	2.749E-14	4.241E-17	3.717E-18	4.443E-10	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F8	p-val	1.180E-12	3.304E-18	3.138E-13	3.304E-18						
	rank	+	+	+	+	+	+	+	+	+	+
F9	p-val	3.160E-09	3.304E-18	2.214E-13	3.428E-09	3.717E-18	2.400E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	-	+	+	+	+
F10	p-val	1	3.304E-18	4.987E-18	3.304E-18	3.381E-14	3.304E-18	1.326E-14	1.080E-09	1	3.304E-18
	rank	≈	+	+	+	+	+	+	+	≈	+
F11	p-val	1.326E-14	3.304E-18	1.508E-17	3.304E-18	3.304E-18	1.812E-13	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F12	p-val	4.181E-18	3.304E-18	3.504E-18	3.304E-18						
	rank	+	+	+	+	+	+	+	+	+	+
F13	p-val	7.048E-15	3.304E-18	1.741E-16	3.504E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F14	p-val	2.208E-12	3.304E-18	7.073E-17	1	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	≈	+	+	+	+	+	+
F15	p-val	3.783E-17	3.304E-18	9.649E-16	8.301E-12	3.304E-18	3.504E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F16	p-val	2.789E-04	3.304E-18	2.656E-03	7.802E-04	1.267E-15	1.823E-12	3.304E-18	3.304E-18	1.268E-17	2.021E-02
	rank	+	+	+	+	-	+	+	+	+	+

Table 13 continue ...

BP		GPE	GPEae	GPEed	LPE	PSO	DE	ABC	GWO	GA	TLBO
F17	p-val	1.496E-08	3.304E-18	2.715E-03	6.451E-03	5.894E-16	1	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	\approx	+	+	+	+
F18	p-val	3.304E-18	3.304E-18	3.304E-18	4.434E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F19	p-val	1.258E-14	3.304E-18	5.031E-17	1.490E-05	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	-	+	+	+	+	+	+
F20	p-val	2.767E-02	3.304E-18	1.131E-05	1.786E-07	1.331E-09	2.878E-05	3.717E-18	7.963E-18	5.894E-16	1.909E-11
	rank	+	+	-	-	+	-	+	+	+	+
F21	p-val	3.015E-11	3.304E-18	1.076E-15	3.717E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F22	p-val	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.302E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	-	+	+	+	+
F23	p-val	4.611E-02	3.304E-18	4.133E-06	9.649E-16	3.304E-18	5.280E-16	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	-	+	+	+	+	+	+	+	+	+
F24	p-val	1	3.304E-18	1.096E-04	1.639E-13	3.304E-18	2.187E-09	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	\approx	+	+	+	+	+	+	+	+	+
F25	p-val	7.485E-17	3.304E-18	3.717E-18	3.304E-18	1.424E-17	2.214E-13	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F26	p-val	1.964E-08	3.304E-18	3.743E-10	3.465E-13	2.176E-15	1.559E-13	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
F27	p-val	2.016E-06	3.304E-18	8.859E-11	8.475E-11	3.304E-18	1.091E-03	3.304E-18	3.304E-18	3.297E-18	3.302E-18
	rank	+	+	+	+	+	+	+	+	+	+
F28	p-val	2.699E-15	3.304E-18	5.031E-17	3.304E-18	5.031E-17	3.052E-05	3.304E-18	3.304E-18	3.304E-18	3.156E-18
	rank	+	+	+	+	+	+	+	+	+	+
F29	p-val	2.768E-10	3.304E-18	4.268E-06	4.544E-03	3.304E-18	8.498E-14	6.304E-18	4.703E-18	1.471E-16	1.150E-11
	rank	+	+	+	+	+	+	+	+	+	+
F30	p-val	1.267E-15	3.304E-18	2.259E-17	5.945E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18	3.304E-18
	rank	+	+	+	+	+	+	+	+	+	+
total '+', symbol		27	30	29	26	29	23	30	30	29	30

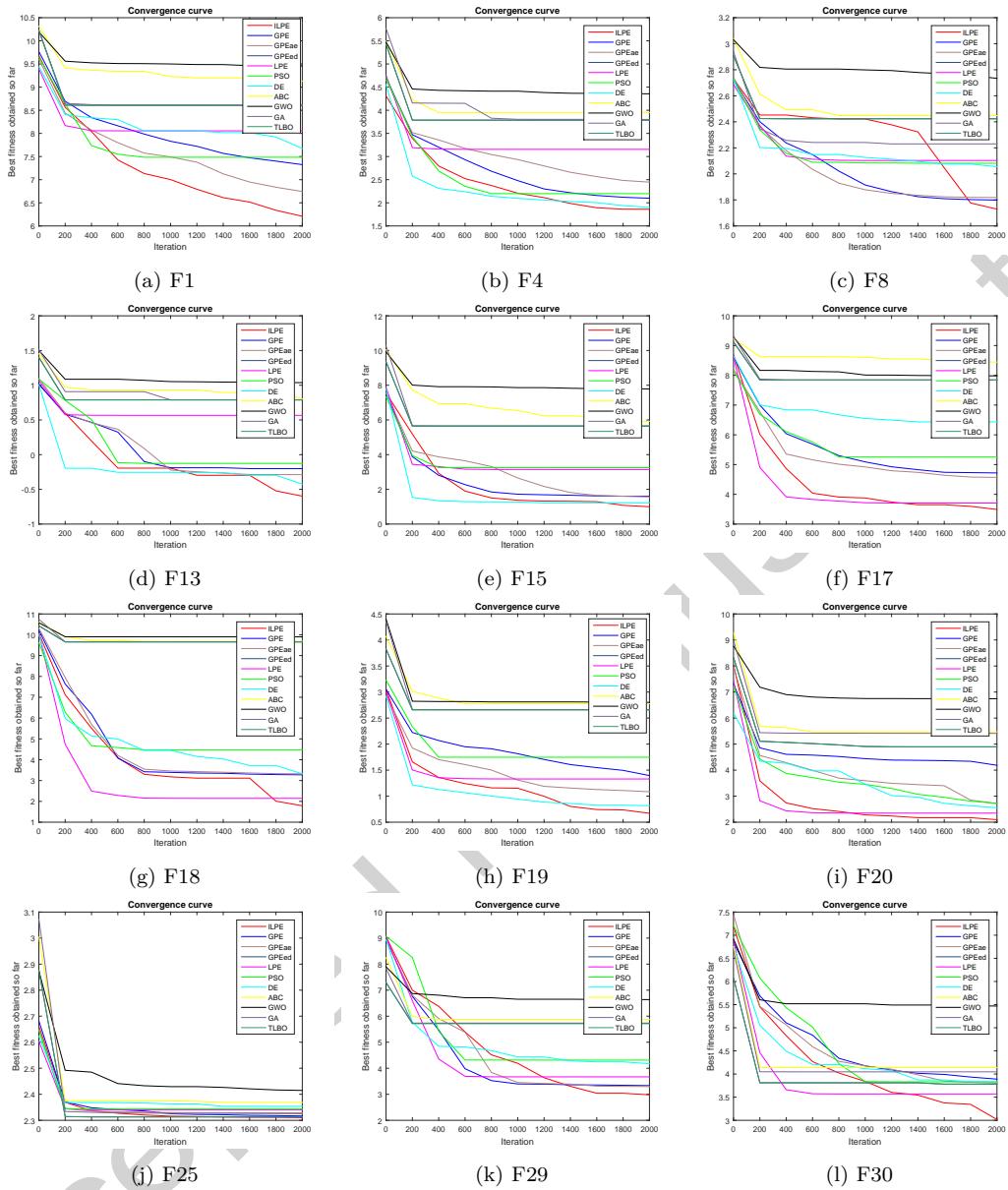


Fig. 1: Convergence curves for CEC2014 benchmark functions

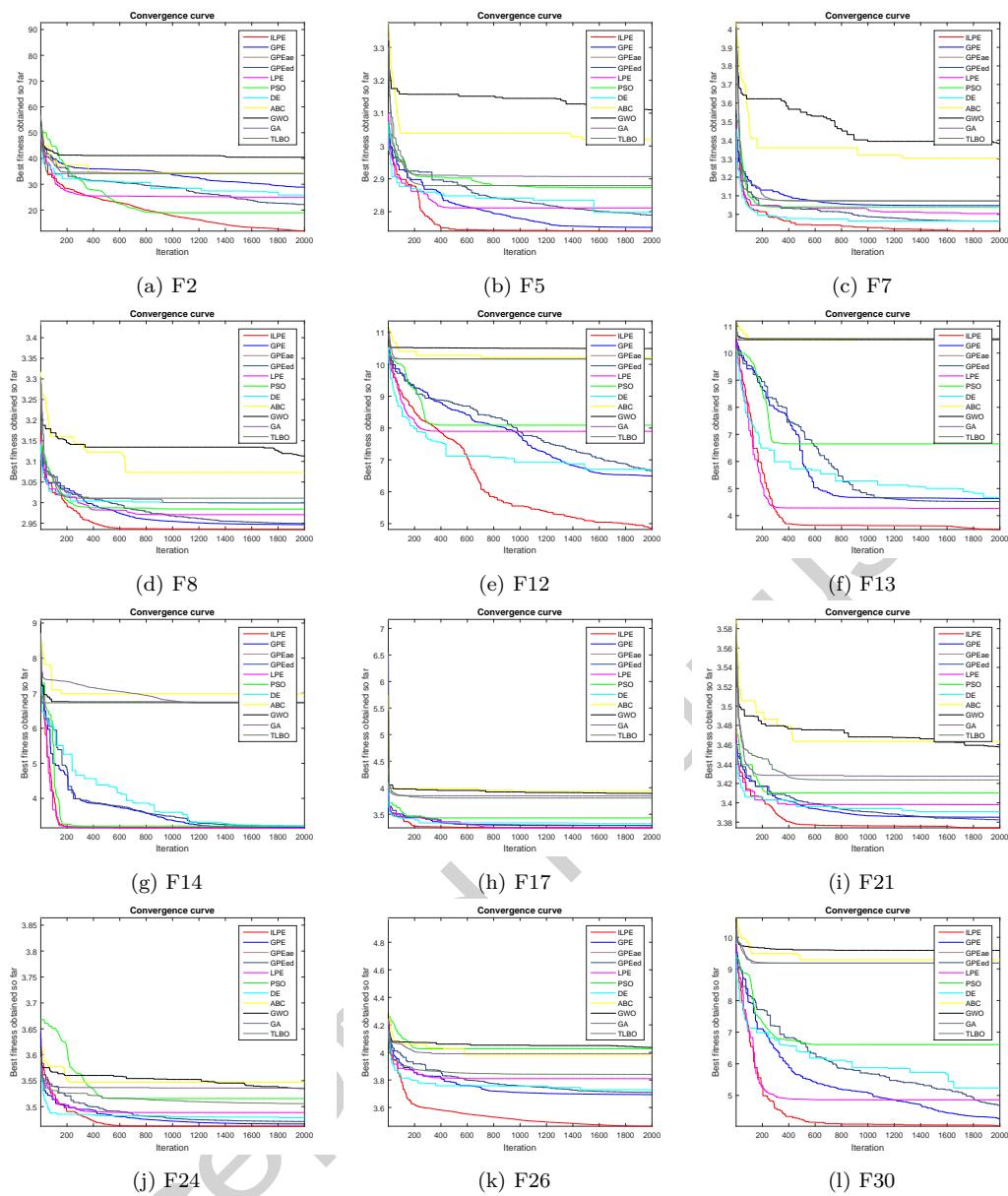


Fig. 2: Convergence curves for CEC2017 benchmark functions

5 Conclusion

LPE is a new and competing evolutionary algorithm with simpler code, less parameters, and extensive search capabilities, which can improve overall performance. This article is an attempt to develop a new strategy called an improved linear prediction algorithm (ILPE). This algorithm treats the population series of an evolutionary algorithm as a time series and then uses a new reproduction operator

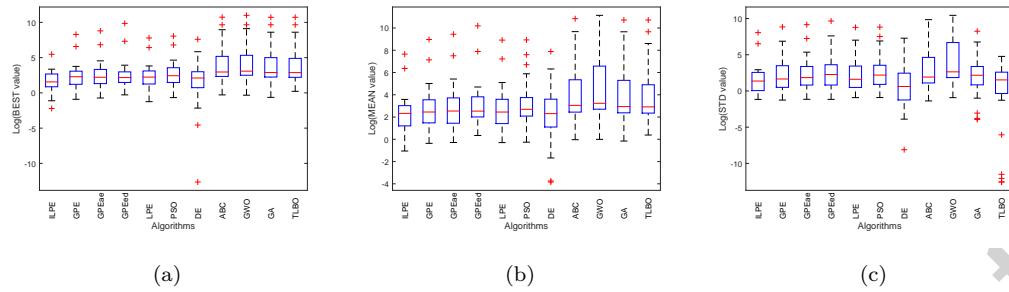


Fig. 3: Box plots for CEC2014 benchmark functions; (a) for BEST, (b) for MEAN, (c) for STD

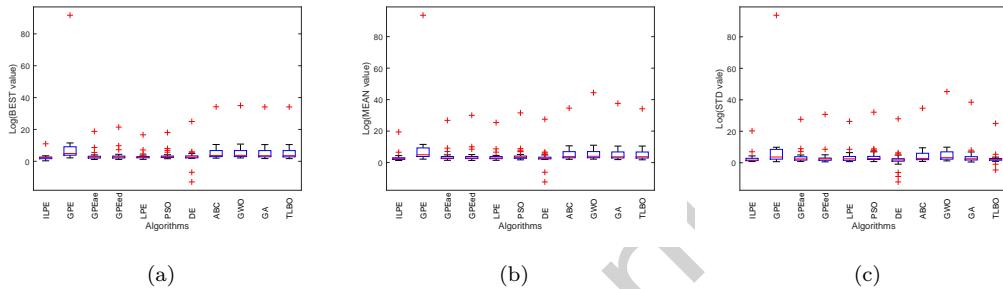


Fig. 4: Box plots for CEC2017 benchmark functions; (a) for BEST, (b) for MEAN, (c) for STD

based on the non-linear least square fitting model to predict the next generation as its offspring. The Proposed ILPE has experimented on CEC2014 and CEC2017 benchmark functions. Results are compared with the original linear prediction evolution algorithm (LPE) and other state-of-the-art metaheuristic algorithms, like GPE, GPEae, GPEed, PSO, DE, GWO, GA, and TLBO algorithms. Statistical analyses revealed that ILPE outperforms original LPE.

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Data availability This manuscript has no associated data.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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