# Fireworks-inspired biogeography-based optimization 

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Published online: 27 June 2018
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#### Abstract

BBO is one of the emerging meta-heuristic optimizer. It is based on sharing the features among islands. This study proposes a hybrid algorithm obtained by incorporating fireworks explosion concept of Fireworks Algorithm into biogeography-based optimization. The hybrid algorithm is named as fireworks-inspired biogeography-based optimization (FBBO). The key feature in the proposed FBBO algorithm is the hybridization of two different searching skills to improve solution quality. FBBO provides a better balance between solution diversification and intensification. The proposed algorithm is tested on CEC 2014 benchmark problems. Numerical experiments demonstrate its effectiveness and accuracy.


Keywords Biogeography-based optimization • Fireworks algorithm • Hybridization • CEC 2014 benchmarks

## 1 Introduction

Computational intelligence is a set of nature-inspired computational methodologies and approaches to solve complex real-world optimization problems. Two paradigms of computational intelligence are evolutionary computation and swarm computation. Now a days evolutionary and swarm computation techniques are a very favorable area for the researchers in the field of numerical optimization. Evolutionary computation is the collection of problem-solving techniques such as evolutionary algorithm, differential evolution and genetic algorithms. These techniques are usually implemented in the computer systems. Evolutionary computation is based on the theory of biological evolution that is used to create optimization procedures to solve complex problems. Evolutionary algorithm is the part of evolutionary computation is inspired by biological evolution such as reproduction, mutation, recombination, natural selection and survival of the fittest. Genetic algorithms, genetic programming, evolutionary programming, and evolutionary strategy are associated with evolutionary algorithm. Swarm computation (swarm optimization or swarm intelligence) is the

[^0]collective behavior of decentralized, self-organized systems. Swarm computation based on the social behavior of organism living in swarms or colonies. In the modern era, we are gaining inspiration from nature. Till the mid-1990s, swarm intelligence approach is considered under evolutionary computation approaches due to their inherent similarities such as the use of population, stochastic nature, application field as well as computer scientists those were familiar with these approaches.

In the literature, BBO has various developments and applications in various fields. The most recent survey on BBO can found in Garg and Deep (2015), Guo et al. (2016) and Ma et al. (2017). BBO is very sensitive to its operators. Migration and mutation are two very crucial operators in BBO. Migration operator is responsible for sharing the information within candidates. The solution quality of candidate highly depends on the migration operator. Mutation operator is responsible to maintain diversity of population. Therefore, in BBO algorithm, there have been lots of developments done by improving the existing operators and by incorporating new operators. Some advanced migration (Ma and Simon 2011; Feng et al. 2014; Xiong et al. 2014; Wang et al. 2014; Farswan and Bansal 2015; Farswan et al. 2016; Garg and Deep 2016; Bansal et al. 2018), mutation (Gong et al. 2010b; Lohokare et al. 2013; Bansal 2016) and new operators (Simon et al. 2014; Bansal and Farswan 2016) are developed earlier. Ma and Simon (2011) proposed Blended $\mathrm{BBO}(\mathrm{BBBO})$ to solving constrained optimization problems. In BBBO , blended information is utilized in migration oper-
ator, i.e., immigrating island accepts the information from itself as well as emigrating island. Feng et al. (2014) proposed an improved BBO (IBBO) using improving migration operator. Xiong et al. (2014) proposed polyphyletic BBO (POLBBO) by incorporating polyphyletic migration operator. Wang et al. (2014) introduced the krill herd algorithm with new migration operator in BBO. Gong et al. (2010b) have applied perturbation in the form of various mutation operators, namely Gaussian mutation, Cauchy mutation and Lévy mutation. Lohokare et al. (2013) proposed a memetic algorithm named as accelerated biogeography-based optimization embedded with a modified differential evolution as a neighborhood search operator (aBBOmDE), for improving convergence speed by modifying mutation operator and maintained exploitation by keeping original migration. Simon et al. (2014) proposed LBBO (linearized BBO) for improving solution of non-separable problems. LBBO combined with periodic re-initialization and local search operator and obtained the algorithm for global optimization in a continuous search space. Bansal and Farswan (2016) proposed DisruptBBO (DBBO) by incorporating a novel disruption operator in BBO algorithm to improve its exploration and exploitation capability.

Further, BBO has applications in various fields such as communication (Ma et al. 2014; Boussaid et al. 2011), image processing (Zhang et al. 2017; Wang et al. 2013b), mechanical engineering and design (Guo et al. 2014), medicine (Rashid et al. 2011), power system (Christy and Raj 2014; Roy et al. 2010; Rarick et al. 2009; Bansal and Farswan 2016) and energy (Niu et al. 2014; Bansal and Farswan 2017; Bansal et al. 2018).

Although, various variants of BBO have developed based on tuning of migration and mutation operators of BBO algorithm. However, this study gives new insights by hybridizing the two meta-heuristic algorithms. Many hybrid NIAs have been proposed to improve performance and to find global optima. The hybridization of the two different searching skills is embedded to improve solution quality. In the literature, recent development of the problem and the algorithm, especially the hybridization of different meta-heuristics, such as hybridization of TLBO (teaching-learning-based optimization) algorithm (Duan et al. 2018), hybridization of local search and global search heuristics (Li et al. 2018), hybridization of ABC (artificial bee colony) and problemspecific heuristic (Li et al. 2016), hybridization of invasive weed optimization (Zheng and Li 2018), and the realistic problem as a hybrid flow shop (HFS) scheduling problem is solved using an effective fruit fly optimization algorithm (FOA) (Li et al. 2014) algorithm.

In the literature, BBO is hybridized with several metaheuristic algorithms given in Table 1. The development of new hybrid NIAs and strategies are worthy of further investigation. Current research directions in hybrid NIAs involve
several major areas. The first area is the determination of how to hybridize a given set of NIAs into a single algorithm; that is, how to determine the hybridization strategy. The second area is the determination of which NIAs to combine in a hybrid algorithm. The third area is the application of hybrid NIAs to special types of optimization problems, such as constrained optimization, multiobjective optimization and CEC benchmark problems. The fourth area is the application of hybrid NIAs to real-world optimization problems. The objective of this paper is to address the first, second and third areas; that is, we emphasize the mechanism of hybridization to improve the optimization performance of NIAs. It is described that BBO and FWA techniques have different strategy to search the optimum solution. Researchers are improving the effectiveness of BBO algorithm. There are various developments done in BBO algorithm. Hybridization of algorithms to improve the quality of solution is new insight of research. In the literature, BBO is hybridized with PSO, DE and ABC etc. There is still scope to develop BBO algorithm by hybridizing with other optimization algorithms. In this study, BBO strategy is influenced by the fireworks explosion strategy (given in Sect. 4).

Rest of the paper is organized as follows: Sect. 2 describes the basic BBO. The brief introduction of fireworks algorithm is given in Sect. 3. Section 4 describes the proposed fireworks-inspired biogeography-based optimization algorithm. Numerical experiments and discussion are given in Sect. 5. Section 6 concludes the paper.

## 2 Biogeography-based optimization

Biogeography is the study of geographical distribution of biological organism over space and time. Robert Mac Arther and Edward Wilson have modeled the mathematical model of biogeography. This model is based on three components such as migration of species, the extinction of existing species and the arrival of new species (MacArthur and Wilson 1967). However, very recently a new evolutionary population-based optimization technique has been proposed which is based on the basic nature of biogeography. It has been named biogeography-based optimization (BBO) (Simon 2008). BBO technique is the inspiration from migration of species within islands (MacArthur and Wilson 1967). BBO procedure has been used to design a population-based optimization procedure that can be potentially applied to optimize many engineering optimization problems.

In biogeography model, the fitness of a geographical area is assessed by habitat suitability index (called $H S I$ ). Habitats which are more favorable and suitable for species to reside are said to have high $H S I$. Similarly, habitats which are less suitable for species to reside are said to have low $H S I$. In this way, high HSI habitats house a relatively larger num-

Table 1 Different hybridization with BBO and its applications

| Researchers | Hybrid algorithms | Applications | Years |
| :---: | :---: | :---: | :---: |
| Zhang et al. (2018) | $\mathrm{BBO}+$ Intuitionistic fuzzy entropy weight method | QoS-aware manufacturing service supply chain optimization | 2018 |
| Yogesh et al. (2017) | $\mathrm{BBO}+\mathrm{PSO}$ | Optimization for emotion and stress recognition from speech signal | 2017 |
| Lim et al. (2016) | BBO + Tabu search | Quadratic assignment problem | 2016 |
| Guo et al. (2014) | $\mathrm{BBO}+\mathrm{PSO}$ | Engineering optimization | 2014 |
| Savsani et al. (2014) | $\mathrm{BBO}+\mathrm{AIA}, \mathrm{ACO}$ | Constrained problems | 2014 |
| Zheng et al. (2014) | $\mathrm{BBO}+\mathrm{DE}$ | Railway wagon scheduling | 2014 |
| Wang et al. (2013a) | $\mathrm{BBO}+\mathrm{HS}$ | Global numerical optimization | 2013 |
| Venkata Rao and Savsani (2012) | $\mathrm{BBO}+\mathrm{ABC}$ | Mechanical design problem | 2012 |
| Boussaid et al. (2011) | $\mathrm{BBO}+\mathrm{DE}$ | Optimal power allocation in wireless sensor networks | 2011 |
| Wang and Ye (2011) | $\begin{aligned} & \mathrm{BBO}+\mathrm{DE}+\text { Simplex } \\ & \text { search } \end{aligned}$ | Parameter estimation of chaotic systems | 2011 |
| Boussaïd et al. (2011) | $\mathrm{BBO}+\mathrm{DE}$ | Standard set of benchmark problems | 2011 |
| Bhattacharya and Chattopadhyay (2011) | $\mathrm{BBO}+\mathrm{DE}$ | Economic load dispatch problem | 2011 |
| Gong et al. (2010a) | $\mathrm{BBO}+\mathrm{DE}$ | Unconstrained problems | 2010 |
| Kundra and Sood (2010) | $\mathrm{BBO}+\mathrm{PSO}$ | Cross-country path planning | 2010 |
| Du et al. (2009) | $\mathrm{BBO}+\mathrm{ES}$, Immigration | Well-known benchmark problems | 2009 |

ber of species. The characterization of habitability are called suitability index variables. Rainfall, vegetation, temperature, etc., are called suitability index variables ( $S I V s$ ). These variables decide or characterize the fitness or $H S I$ of a solution. In BBO model, two parameters, immigration rate $(\lambda)$ and emigration rate $(\mu)$ governs the migration of species from one habitat to another habitat. Here both $\lambda$ and $\mu$ depend on the number of species in a habitat. The relation between migration rate (immigration rate $\lambda$ and emigration rate $\mu$ ) and the number of species is illustrated in Fig. 1. If there are zero species on the island, then immigration rate is maximum, denoted by $I$. If there are maximum number of species $\left(S_{\max }\right)$ on the island, then emigration rate is maximum, denoted by $E$. At the state of equilibrium, the number of species is denoted by $S_{0}$ and in equilibrium state, immigration rate and emigration rate are equal. The islands are referred to as high $H S I$ islands if the number of species is above than $S_{0}$ and the islands are referred to as low $H S I$ island if the number of species is less than $S_{0}$. Further, mathematical model of species counts in biogeography is as follows.

Let us assume that $P_{s}(t)$ is the probability given for $s$ species in the habitat at any time $t$.


Fig. 1 Relation between number of species and migration rate. Reproduced with permission fromSimon (2008)

$$
\begin{align*}
P_{s}(t+\Delta t)= & P_{s}(t)\left(1-\lambda_{s} \Delta t-\mu_{s} \Delta t\right) \\
& +P_{s-1} \lambda_{s-1} \Delta t+P_{s+1} \mu_{s+1} \Delta t \tag{1}
\end{align*}
$$

where $\lambda_{s}$ is immigration rate when there are $s$ species in the habitat. $\mu_{s}$ is emigration rate when there are $s$ species in the habitat.

At time $t+\Delta t$, one of the following conditions must hold for $s$ species in the habitat.

1. If there are $s$ species in the habitat at time $t$, then there will be no immigration and no emigration of species within time $t$ and $t+\Delta t$.
2. If there are $(s-1)$ species in the habitat at time $t$ then one species will immigrate between time $t$ and $t+\Delta t$.
3. If there are $(s+1)$ species in the habitat at time $t$, then one species will emigrate between time $t$ and $t+\Delta t$.

For ignoring the probability of more than one immigration or emigration, $\Delta t$ is assumed to be very small. Taking limit as $\Delta t \longrightarrow 0$
$\dot{P}_{s}= \begin{cases}-\left(\lambda_{s}+\mu_{s}\right) P_{s}+\mu_{s+1} P_{s+1}, & s=0 \\ -\left(\lambda_{s}+\mu_{s}\right) P_{s}+\lambda_{s-1} P_{s-1}+\mu_{s+1} P_{s+1}, & 1 \leq s \leq s_{\max }-1 \\ -\left(\lambda_{s}+\mu_{s}\right) P_{s}+\lambda_{s-1} P_{s-1}, & s=s_{\text {max }}\end{cases}$

Let us define $\lambda_{n}$ is maximum immigration and $\mu_{n}$ is maximum emigration rate. Maximum possible number of species in the habitat is $S_{\max }$. Therefore, we can obtain a matrix relation exhibiting the dynamic equations of the probabilities of the number of species in the habitat as:

$$
\begin{align*}
& {\left[\begin{array}{c}
\dot{P}_{0} \\
\dot{P}_{1} \\
\vdots \\
\vdots \\
\dot{P}_{S_{\max }}
\end{array}\right]} \\
& =\left[\begin{array}{ccccc}
-\left(\lambda_{0}+\mu_{0}\right) & \mu_{1} & 0 & \cdots & 0 \\
\lambda_{0} & -\left(\lambda_{1}+\mu_{1}\right) & \mu_{2} & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \lambda_{n-2}-\left(\lambda_{n-1}+\mu_{n-1}\right) & \mu_{n} \\
0 & \cdots & 0 & \lambda_{n-1} & -\left(\lambda_{n}+\mu_{n}\right)
\end{array}\right] \\
& \quad \times\left[\begin{array}{c}
P_{0} \\
P_{1} \\
\vdots \\
\vdots \\
P_{S_{\max }}
\end{array}\right] \tag{3}
\end{align*}
$$

In BBO procedure, two simple biogeography concepts migration and mutation are present.

In the designed BBO algorithm, each habitat $H$ has a potential $m \times 1$ vector solution where $m$ is the number of

SIVs in each habitat. HSI of each habitat corresponds to fitness function of population-based algorithms. Habitat with the highest HSI reveals the best candidate for the optimum solution among all habitats. It is assumed that the ecosystem constitutes $N_{p}$ habitats, i.e., the population size is $N_{p}$. In the basic BBO algorithm, the immigration and emigration rates are calculated using the following formulae:
$\lambda_{i}=I\left(1-\frac{k_{i}}{n}\right)$
$\mu_{i}=E\left(\frac{k_{i}}{n}\right)$
here $\lambda_{i}$ is immigration rate and $\mu_{i}$ is the emigration rate of the $i^{t h}$ candidate (habitat), $n$ is the maximum possible number of species in a habitat. The fitness rank of $i^{t h}$ habitat is $k_{i}$ (after sorting the habitat based on fitness value). Therefore, rank 1 and $n$ for worst and the best solution, respectively.

The best solution remains in the competition using elitism operator in population-based optimization algorithms. The usage of elitism operator in BBO is to prevent the best solution from corruption. In elitism approach, we save the features of the best habitat. Elitism can be implemented by setting $\lambda=0$ for $p$ best habitats. Here $p$ is elitism parameter selected by the user.
The pseudocode of BBO is as follows in Algorithm 1:

```
Algorithm 1 Biogeography-based optimization algorithm
    Initialize the population.
    Sort the population in descending order of fitnesses.
    Calculate \(\lambda_{i}\) and \(\mu_{e} \forall i, e \in 1,2,3, \ldots ., N_{p}\).
    for Generation index \(=1\) to Maximum generation do
        \(\backslash \backslash\) Apply the migration operator
        for \(i=1\) to \(N_{p}\) do
            Select habitat \(H_{i}\) according to \(\lambda_{i}\).
            if \(\operatorname{rand}(0,1)<\lambda_{i}\) then
                for \(e=1\) to \(N_{p}\) do
                    Select habitat \(H_{e}\) according to \(\mu_{e}\).
                    Replace the selected SIV of \(H_{i}\) by randomly selected
                    SIV of \(H_{e}\).
                end for
            end if
        end for
        \(\backslash \backslash\) Apply the mutation operator
        for \(i=1\) to \(N_{p}\) do
            Compute mutation probability \(m(S)\).
            if \(\operatorname{rand}(0,1)<m(S)\) then
                Replace \(H_{i}(S I V)\) with randomly generated SIV.
            end if
        end for
        Sort the population in descending order of fitnesses.
        \(\ \backslash\) Apply elitism
        Save some (elitism size) best solution of previous generation in
        current solution.
        Stop, if termination criterion is satisfied.
    end for
```

Fig. 2 Fireworks explosion. [Credit: Google Images]


Migration and mutation are two crucial operators in BBO. Migration and the mutation procedures make possible to evolve new candidate solutions. This procedure of governing the habitats to the migration procedure, followed by the mutation procedure, is continued to next generation until the termination criteria are satisfied. These criteria can be the maximum number of generations or obtaining the desired solution. Migration operator of BBO is responsible for sharing the information within habitats using their migration rates ( $\lambda$ and $\mu$ ). The migration operator is same as the crossover operator of the evolutionary algorithm and is responsible for sharing the features among candidate solutions for modifying fitness. In the migration procedure, immigrating habitat is selected according to the probability of immigration rate and emigrating habitat is selected according to the probability of emigration rate of habitats. The individual solution vector is depicted through $S I V s$ in BBO. Then it is probabilistically decided that to which of the $S I V$ of immigrating habitat needs to be modified. Once the $S I V$ is selected, algorithm replaces that $S I V$ by emigrating habitat's SIV .

The other crucial operator is mutation operator. Mutation operator in BBO is modeled as $S I V$ mutation. In many meta-heuristics, mutation rate is predefined by users and in entire optimization process the value is fixed. This operator is responsible to maintain diversity of population in BBO procedure.

The probability of each species count can be produced via migration model shown in Fig. 1. By observing the equilibrium point of species curve, we can conclude that both low and high species count have relatively low probabilities. Both very high and very low HSI solution are improbable at the same rate. Medium HSI solution is relatively probable. Therefore, mutation process gives same chance to improve low HSI solutions as to high HSI solutions. The mutation rate $\operatorname{mut}(i)$ is calculated as:
$\operatorname{mut}(i)=m_{\max }\left(1-\frac{P_{i}}{P_{\max }}\right)$
where $m_{\max }$ is the user defined parameter and $P_{\max }=$ $\max \left\{P_{i}\right\} ; i=1,2, \ldots, N_{p}$.

## 3 Fireworks algorithm

Fireworks Algorithm (FWA) is inspired by the explosion process of fireworks (given in Fig. 2). FWA was proposed by Tan and Zhu (2010). The algorithm starts with random locations of individuals in the search space. Each location explodes a firework to produce a set of sparks. In the next iteration, only high-quality fireworks are selected among firework and sparks. Quality of fireworks improved until the termination criterion is met.

If the given optimization problem is $\operatorname{Min} f(X)$, where $X=\left(x^{1}, \ldots, x^{D}\right)$ and $x_{\text {min }}^{j} \leq x_{i}^{j} \leq x_{\text {max }}^{j}, \forall j=1, \ldots, D$ and $\forall i=1, \ldots, N_{p}$. The number of sparks corresponding to each firework $X_{i}$ is defined as follows:
$s_{i}=m \cdot \frac{f_{\text {worst }}-f\left(X_{i}\right)+\epsilon}{\sum_{i=1}^{N_{p}}\left(f_{\text {worst }}-f\left(X_{i}\right)\right)+\epsilon}$
where parameter $m$ controls the total number of sparks generated by the $N_{p}$ fireworks, $f_{\text {worst }}$ is the worst (maximum) value of objective function among $N_{p}$ fireworks and $\epsilon$ is the smallest constant to avoid zero-division error.

To remove overwhelming effects of splendid fireworks, the bounds of sparks are defined as follows:
$\hat{s_{i}}=\left\{\begin{array}{lc}\operatorname{round}(\text { a.m) } & \text { if } s_{i}<a m \\ \operatorname{round}(b . m) & \text { if } s_{i}>b m \\ \operatorname{round}\left(s_{i}\right) & \text { otherwise }\end{array}\right.$
where $a$ and $b$ are constant parameters, $\hat{s_{i}}$ is the bound of sparks and round() is the rounding function.

The fireworks with better quality have a lower explosion amplitude and vice versa. For each firework $X_{i}$, the explosion amplitude $A_{i}$ is defined as follows:
$A_{i}=\hat{A} \cdot \frac{f\left(X_{i}\right)-f_{\text {best }}+\epsilon}{\sum_{i=1}^{N_{p}}\left(f\left(X_{i}\right)-f_{\text {best }}\right)+\epsilon}$
where $\hat{A}$ is constant and is calculated as the sum of all amplitudes and $f_{\text {best }}$ is the best (minimum) value of objective function among $N_{p}$ fireworks.

Firstly, initialize the location of sparks around each firework as:
$\tilde{X}_{j}=X_{i}, \forall j \in 1,2, \ldots, s_{i}$, for each firework $i \in 1,2, \ldots, N_{p}$

Then the location of sparks are updated using an update equation. The update equation is based on displacement factor $h=A_{i} \cdot \operatorname{rand}(-1,1)$ in the firework location $X_{i}, 1 \leq i \leq N_{p}$. The location of each spark $\tilde{X}_{j}, 1 \leq j \leq s_{i}$ is calculated as follows:
$\tilde{X_{j}^{d}}=\tilde{X_{j}^{d}}+h$,
where $d$ is randomly selected dimension.
If the location of sparks fall out of the search space, then it is mapped into the search space as below:
$\tilde{X}_{j}^{d}=X_{\min }^{d}+\operatorname{round}\left(\tilde{X_{j}^{d}}\right) \bmod \left(X_{\max }^{\tilde{d}}-X_{\min }^{\tilde{d}}\right)$
where $\tilde{X}_{j}^{d}$ is the position of an spark which lies outside of the search space and $X_{\text {min }}^{\tilde{d}}$ and $X_{\text {max }}^{\tilde{d}}$ are the boundaries of spark $\tilde{X}$ in the direction $d$. Also "mod" is the modulo operator.

Then the Gaussian mutation is applied over sparks to preserves the diversity of sparks. Then the location of specific spark $\hat{X}_{j}, 1 \leq j \leq s_{i}$ is calculated as follows:
$\hat{X_{j}^{d}}=\hat{X_{j}^{d}} \cdot g$
where $g$ is Gaussian coefficient (Gaussian $(1,1)$ ). $\operatorname{Gaussian}(1,1)$ is normally distributed random number with mean $\mu=1$ and variance $\sigma=1$ and $d$ is a randomly selected dimension.

If the location of specific sparks fall out of the search space, then it is mapped into the search space as below:
$\hat{X_{j}^{d}}=X_{\min }^{d}+\operatorname{round}\left(\hat{X}_{j}^{d}\right) \bmod \left(X_{\max }^{\hat{d}}-X_{\min }^{\hat{d}}\right)$
where $\hat{X}_{j}^{d}$ is the position of an specific spark which lies outside of the search space and $X_{\text {min }}^{\hat{d}}$ and $X_{\text {max }}^{\hat{d}}$ are the boundaries of specific spark $\hat{X}$ in the direction $d$. Also "mod" is the modulo operator.

In each iteration, the best location among all current sparks are always selected for next iteration. In selection mechanism, the measurement of Euclidean distance is applied, where $d\left(X_{i}, X_{j}\right)$ represents the Euclidean distance between any two individuals $X_{i}$ and $X_{j}$.
$R\left(X_{i}\right)=\sum_{j \in K} d\left(X_{i}, X_{j}\right)=\sum_{j \in K}\left\|X_{i}-X_{j}\right\|$
where $R\left(X_{i}\right)$ represents the sum of distances between individual $X_{i}$ and all the other individuals. $K$ denotes the set of all current locations of sparks after explosion operator and Gaussian mutation operator. The selection probability of the location $X_{i}$ for next generation is calculated from the roulette wheel selection mechanism:

$$
\begin{equation*}
p\left(X_{i}\right)=\frac{R\left(X_{i}\right)}{\sum_{j \in K} R\left(X_{j}\right)} \tag{15}
\end{equation*}
$$

Working of Fireworks Algorithm is given in Algorithm 2.

```
Algorithm 2 Fireworks algorithm
    Initialize the location of fireworks.
    for Generation index \(=1\) to Maximum generation do
        Calculate \(s_{i}\) using equations (7) and (8), \(\forall i \in 1,2,3, \ldots ., N_{p}\).
        Calculate \(A_{i}\) using Equation (9), \(\forall i \in 1,2,3, \ldots ., N_{p}\).
        \(z=\operatorname{round}(\operatorname{D} \cdot \operatorname{rand}(0,1)) \backslash \backslash\) randomly choose \(z\) dimension.
        Calculate displacement factor, \(h=A_{i} \cdot \operatorname{rand}(-1,1)\)
        for \(j=1\) to \(s_{i}\) do
            for \(d=1\) to \(D\) do
            if \(d \in z\) then
                        \(\tilde{X_{j}^{d}}=\tilde{X_{j}^{d}}+h\)
            end if
            end for
        end for
        \(\ \backslash\) Find the location of specific sparks
        \(z=\operatorname{round}(D \cdot \operatorname{rand}(0,1)) \backslash \backslash\) randomly choose \(z\) dimension.
        Calculate Gaussian explosion coefficient, \(g=\operatorname{Gaussian}(1,1)\);
        for \(j=1\) to \(s_{i}\) do
            for \(d=1\) to \(D\) do
            if \(d \in z\) then
                    \(\hat{X_{j}^{d}}=\hat{X_{j}^{d}} \cdot g\)
            end if
            end for
        end for
        Select best location among current locations of fireworks and
        sparks.
        Select \(N_{p}-1\) location according to selection probability given in
        Equation (15).
        Stop, if termination criterion is satisfied.
    end for
```


## 4 Proposed method

### 4.1 Motivation

Hybrid nature-inspired algorithms (NIAs) are attractive alternatives to standard NIAs. The combination of several algorithms in hybrid NIAs allows it to exploit the strength of each algorithm. It has been shown that by properly selecting the constituent algorithms and hybridization strategies, hybrid NIAs can outperform their constituent algorithms due to their synergy. This characteristic is strong motivation for the study of hybrid NIAs. Researchers are continuously developing more promising and refined nature-inspired algorithms by acquiring the different search techniques in one specific optimization framework. In this paper, BBO and FWA are considered for hybridization as BBO (Bansal and Farswan 2017; Bansal et al. 2018) has proven a good optimizer and FWA is attracted by their sparks generation skill. Initially, individuals are updated by BBO strategy and followed by the strategy which is inspired by fireworks explosion (given in Sect. 3).

### 4.2 Fireworks-inspired biogeography-based optimization algorithm

Each meta-heuristic algorithm has its own exploration and exploitation capability to search the promising solution in the search space. The hybridization of BBO and FWA is based on different exploration and exploitation capability of algorithms. BBO has migration operator to share the existing features and mutation operator to keep the diversity. FWA has explosion phenomena to improve the solution. In FWA, each firework generate the set of sparks and some specific sparks is generated by Gaussian distribution to keep diversity. In the proposed method both BBO and FWA algorithms are hybridized together to produce better optimal solution. The working of proposed hybrid approach is as follows:

Since we have three population: before applying BBO operators, after applying BBO operators, and after applying strategy inspired by fireworks explosion. Let us call these population as parents, leaders and followers, respectively. Initially population is generated within the search space called parents. Then BBO operators are applied within the parent individuals. BBO operators are responsible to produce leaders. Each leader generates some individuals named as followers. The generation of followers through each leader is inspired by fireworks explosions described in Sect. 3. In the proposed algorithm, leaders corresponds to the fireworks and followers generated by each leader corresponds to sparks generated by related firework. The range of followers is determined by Eq. (9), and the number of followers corresponding to each leader is calculated by Eqs. (7) and (8). The location of all followers corresponding to each leader are determined
by Eqs. (10), (12) and (11). Leaders are strongly connected to those followers which are settled in closed vicinity of leaders. The good leader denotes that the promising area of leader may be closed to the optimal location. Thus it is proper to utilize the more followers to search the local area around the leader. In another way, a bad leader means the optimal location may be apart from the location of leader. Then, the search range should be larger. In FBBO, more followers are generated and the location range of the followers is smaller for good leader, compared to bad one.

From the leaders and followers, $n-1$ individuals are selected based on the selection probability given in Eq. (15) as well as 1 best individual is selected to proceed in the next step. Then elitism is applied in population. Elitism operator saves the two best individual in each generation. The pseudocode of proposed algorithm is as follows:

```
Algorithm 3 Hybrid BBO and FWA algorithm
    Initialize \(N_{p}\) locations (individuals).
    for Generation index \(=1\) to Maximum generation do
        Apply BBO search mechanism
        Apply FWA search mechanism
        Apply elitism
        Stop, if termination criterion is satisfied.
    end for
```


### 4.3 Evaluating FBBO for bias(es)

It is better to obtain an idea of optimizer's intrinsic bias(es) before evaluating the performance of an optimizer using numerical experiments on benchmark set. The nature of optimizers may have central bias (increased possibility to search solutions near to the center of the search space) and/or an edge bias (increased possibility to search solutions near to the edges of the search space) and/or axial bias (increased possibility to search solutions along a coordinate axis and variation of this bias also increased possibility to search solution along a diagonal of the search space) and/or exploitation bias (increased possibility to search solutions around a position with no special characteristics). Therefore, to test FBBO and other considered algorithms for bias(es), signature analysis (Clerc 2015) has been carried out. Let us consider the minimization problem:

$$
\operatorname{Minf}\left(x_{1}, x_{2}\right)=5 ; x_{1}, x_{2} \in[-5,5]
$$

Clearly, every point in the search space is an optimal solution of the problem. Therefore, an unbiased optimizer should provide the solution same as random search. Signatures for BBO, M1BBO, M2BBO, DBBO, LBBO, BBBO, and FBBO are plotted in Fig. 3a-g, respectively. In these signatures, solutions obtained by an algorithm in 100 runs having 1000


Fig. 3 Analysis based on signatures. a Signature of BBO algorithm, $\mathbf{b}$ signature of M1BBO algorithm, $\mathbf{c}$ signature of M2BBO algorithm, d signature of DBBO algorithm, $\mathbf{e}$ signature of LBBO algorithm, $\mathbf{f}$ signature of BBBO algorithm and $\mathbf{g}$ signature of FBBO algorithm
iterations in each run are plotted. Detailed parameter settings of these algorithms are given in Sect. 5.1. From the signatures, it is clear that LBBO and BBBO are central bias algorithms. That is, LBBO and BBBO are a better algorithm for those problems whose optima lies in the close vicinity of the center of the search space. The original BBO, M1BBO, M2BBO, DBBO, LBBO, and proposed FBBO are almost unbiased algorithms. That is the location of the optima will have the least impact on the performance of $\mathrm{BBO}, \mathrm{M} 1 \mathrm{BBO}$, M2BBO, DBBO, LBBO, and FBBO.

## 5 Experimental results and discussion

To see the effect of fireworks-inspired biogeography-based optimization (FBBO) on CEC 2014 (Liang et al. 2013) benchmark set is selected for experiments. This set of problems consists unimodal, multimodal, hybrid and composite functions.

### 5.1 Experimental setting

The proposed FBBO is tested for 10 and 30 -dimensional search space. 51 independent runs are conducted for each function. The search space range is [-100,100]. Initial population is uniformly generated within the specified search range using random number generator based on clock time. The population size is set 5 for the proposed FBBO algorithm otherwise population size is considered 50. The termination criteria is either maximum function evaluation $\left(10^{4} \times\right.$ Dimension) or error value with desired level of accuracy $\left(10^{-8}\right)$, whichever is attained earlier. Other parameter settings for the algorithms BBO, GSA, and DBBO are similar to their original research papers.

### 5.2 Analysis of results

In this paper, results are reported in the format as required by CEC 2014. Table 2 shows the results obtained by performing GSA, BBO, M1BBO, M2BBO, DBBO, LBBO, BBBO and FBBO on the basis of these benchmark function for 10 -dimensional search space. Table 3 gives the results obtained after performing the same experiments on functions of 30-dimensional search space. The performance of FBBO is compared with GSA, $\mathrm{BBO}, \mathrm{M} 1 \mathrm{BBO}, \mathrm{M} 2 \mathrm{BBO}, \mathrm{DBBO}$, LBBO, and BBBO. The recorded results are the minimum, maximum, mean, median, and standard deviation of the error value of different 51 runs. The error is the absolute value of difference between obtained objective function value and the known function value. The tabulated data of the results is presented as instructed in the directions in the problem set. In Table 2, FBBO is compared with other algorithm and variants of BBO . The performance of FBBO is analyzed
based on the reported results. The better results obtained by considered algorithms are highlighted by bold font. In case of average error value FBBO performs better except 9 functions ( $f_{8}, f_{9}, f_{12}, f_{14}, f_{17}, f_{20}, f_{24}, f_{29}, f_{30}$ ). For the function $f_{22}$, M1BBO and FBBO have same average error value which are minimum corresponding to other considered algorithms. In the function $f_{26}$, all considered algorithms have minimum and equal average error value except GSA. Out of the 30 functions FBBO is better on 10, 15, and 12 functions based on standard deviation, median and worst error value, respectively. FBBO gives better minimum value in all function except $f_{8}, f_{9}, f_{10}, f_{12}, f_{20}, f_{22}, f_{24}, f_{25}$ and $f_{27}$. In all aspects given in Table 2, FBBO is performing better than considered algorithms. Same analysis is carried out for 30-dimensional CEC 2014 functions. In the Table 3, better average error achieved by FBBO in 22 functions out of total 30 CEC 2014 functions. FBBO performs better on 30 -dimensional space except 8 functions $\left(f_{7}, f_{8}, f_{9}, f_{12}, f_{14}, f_{18}, f_{19}, f_{21}\right)$. Based on standard deviation, median and worst error value FBBO is performing better on 14,16 , and 15 functions, respectively. The better minimum value achieved by FBBO in 15 functions as compared to other considered algorithms. For the function $f_{5}$ all considered algorithms have same minimum values. From the above discussion, we can conclude that FBBO is better than considered algorithms for achieving the set target given by CEC 2014. Thus, in order to attain a better objective value, FBBO is preferred over considered algorithms.

Comparison of the proposed FBBO with state-of-the-art algorithms such as FOA (Li et al. 2014), ABC (Li et al. 2016), TLBO (Duan et al. 2018) and invasive weed optimization (Zheng and Li 2018) may be a matter of future research.

Some more intensive statistical analyses have been carried out with the numerical results of GSA, BBO, M1BBO, M2BBO, DBBO, LBBO, BBBO and FBBO. The boxplots are the empirical distribution of data. In 10-dimensional search space, boxplots for mean error, standard deviation, median, best and worst corresponding to all algorithms GSA, BBO, M1BBO, M2BBO, DBBO, LBBO, BBBO, and FBBO are given in Fig. 4. The boxplot for the same in 30dimensional search space is given in Fig. 5. From the boxplot analyses, FBBO performs better than other considered algorithms for $10-$ and 30 -dimensional problems.

### 5.3 Statistical analysis

In this section, Mann-Whitney U rank-sum test used to analyze the significance difference between FBBO and other considered algorithms. The results of Mann-Whitney U rank-sum test for minimum error of 100 simulations are given in Tables 4 and 5 for 10 -dimensional and 30 -dimensional, respectively. In Tables 4 and 5, ' + ' sign appears if FBBO is the better algorithm, '-' sign appears if FBBO is the worse

Table 2 Average, standard deviation, median, best, worst error value obtained by FBBO and other variant of BBO for 10-dimensional CEC 2014 benchmark problems

| TP | Algorithms | Mean error | SD | Med | Best | Worst |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{1}$ | GSA | $2.70 \mathrm{E}+06$ | $9.20 \mathrm{E}+05$ | $2.63 \mathrm{E}+06$ | $1.45 \mathrm{E}+06$ | $4.07 \mathrm{E}+06$ |
|  | BBO | $5.12 \mathrm{E}+06$ | $6.35 \mathrm{E}+06$ | $2.66 \mathrm{E}+06$ | $6.95 \mathrm{E}+04$ | $3.11 \mathrm{E}+07$ |
|  | M1BBO | $3.63 \mathrm{E}+05$ | 3.59E+05 | $2.75 \mathrm{E}+05$ | $8.08 \mathrm{E}+03$ | $\mathbf{1 . 7 3 E}+06$ |
|  | M2BBO | $3.21 \mathrm{E}+05$ | $6.94 \mathrm{E}+06$ | $4.79 \mathrm{E}+06$ | $3.84 \mathrm{E}+03$ | $2.64 \mathrm{E}+07$ |
|  | DBBO | $2.70 \mathrm{E}+05$ | $5.54 \mathrm{E}+05$ | $1.84 \mathrm{E}+05$ | $2.11 \mathrm{E}+03$ | $3.08 \mathrm{E}+06$ |
|  | LBBO | $2.57 \mathrm{E}+06$ | $2.25 \mathrm{E}+06$ | $2.27 \mathrm{E}+06$ | $1.69 \mathrm{E}+04$ | $8.72 \mathrm{E}+06$ |
|  | BBBO | $3.93 \mathrm{E}+06$ | $2.61 \mathrm{E}+06$ | $3.82 \mathrm{E}+06$ | $3.95 \mathrm{E}+04$ | $1.03 \mathrm{E}+07$ |
|  | FBBO | 1.07E+05 | $4.90 \mathrm{E}+05$ | $1.92 \mathrm{E}+05$ | $1.01 \mathrm{E}+03$ | $2.76 \mathrm{E}+06$ |
| $f_{2}$ | GSA | $2.49 \mathrm{E}+02$ | $4.02 \mathrm{E}+02$ | $1.50 \mathrm{E}+02$ | $7.38 \mathrm{E}-01$ | 1.80E+03 |
|  | BBO | $5.74 \mathrm{E}+04$ | $3.81 \mathrm{E}+04$ | $5.17 \mathrm{E}+04$ | $1.30 \mathrm{E}+04$ | $2.18 \mathrm{E}+05$ |
|  | M1BBO | $2.23 \mathrm{E}+03$ | $2.82 \mathrm{E}+03$ | $8.35 \mathrm{E}+02$ | $2.14 \mathrm{E}+00$ | $1.47 \mathrm{E}+04$ |
|  | M2BBO | $4.87 \mathrm{E}+04$ | $4.12 \mathrm{E}+04$ | $3.28 \mathrm{E}+04$ | $7.00 \mathrm{E}+03$ | $2.15 \mathrm{E}+05$ |
|  | DBBO | $9.39 \mathrm{E}+03$ | $7.53 \mathrm{E}+03$ | $7.29 \mathrm{E}+03$ | $3.69 \mathrm{E}+02$ | $3.91 \mathrm{E}+04$ |
|  | LBBO | $1.03 \mathrm{E}+03$ | $1.23 \mathrm{E}+03$ | $5.07 \mathrm{E}+02$ | $1.80 \mathrm{E}-01$ | $5.65 \mathrm{E}+03$ |
|  | BBBO | $1.35 \mathrm{E}+03$ | $1.15 \mathrm{E}+03$ | $1.13 \mathrm{E}+03$ | $3.42 \mathrm{E}+01$ | $5.59 \mathrm{E}+03$ |
|  | FBBO | 1.15E+01 | $3.85 \mathrm{E}+03$ | $2.15 \mathrm{E}+03$ | $1.54 \mathrm{E}-01$ | $1.39 \mathrm{E}+04$ |
| $f_{3}$ | GSA | $1.90 \mathrm{E}+04$ | $4.33 \mathrm{E}+03$ | $1.94 \mathrm{E}+04$ | $1.13 \mathrm{E}+04$ | $2.75 \mathrm{E}+04$ |
|  | BBO | $9.09 \mathrm{E}+03$ | $7.94 \mathrm{E}+03$ | $6.89 \mathrm{E}+03$ | $1.27 \mathrm{E}+02$ | $3.46 \mathrm{E}+04$ |
|  | M1BBO | $6.50 \mathrm{E}+03$ | $5.40 \mathrm{E}+03$ | $4.85 \mathrm{E}+03$ | $6.22 \mathrm{E}+01$ | $2.34 \mathrm{E}+04$ |
|  | M2BBO | $6.49 \mathrm{E}+03$ | $5.98 \mathrm{E}+03$ | $6.17 \mathrm{E}+03$ | $2.65 \mathrm{E}+02$ | $3.32 \mathrm{E}+04$ |
|  | DBBO | $6.57 \mathrm{E}+03$ | $5.40 \mathrm{E}+03$ | $4.64 \mathrm{E}+03$ | $1.26 \mathrm{E}+02$ | $2.46 \mathrm{E}+04$ |
|  | LBBO | $4.37 \mathrm{E}+03$ | $3.75 \mathrm{E}+03$ | $3.53 \mathrm{E}+03$ | $4.47 \mathrm{E}+00$ | $1.44 \mathrm{E}+04$ |
|  | BBBO | $2.01 \mathrm{E}+03$ | $1.70 \mathrm{E}+03$ | $1.53 \mathrm{E}+03$ | $3.69 \mathrm{E}+01$ | $7.32 \mathrm{E}+03$ |
|  | FBBO | $2.53 \mathrm{E}+02$ | $3.47 \mathrm{E}+03$ | 1.16E+03 | $2.31 \mathrm{E}+00$ | $2.03 \mathrm{E}+04$ |
| $f_{4}$ | GSA | $4.41 \mathrm{E}+01$ | $1.40 \mathrm{E}+01$ | $4.56 \mathrm{E}+01$ | $2.12 \mathrm{E}-01$ | $6.16 \mathrm{E}+01$ |
|  | BBO | $1.69 \mathrm{E}+01$ | $1.65 \mathrm{E}+01$ | $5.26 \mathrm{E}+00$ | $5.30 \mathrm{E}-02$ | $3.58 \mathrm{E}+01$ |
|  | M1BBO | $3.94 \mathrm{E}+00$ | $1.78 \mathrm{E}+00$ | $4.78 \mathrm{E}+00$ | $3.01 \mathrm{E}-05$ | $5.46 \mathrm{E}+00$ |
|  | M2BBO | $1.45 \mathrm{E}+01$ | $1.77 \mathrm{E}+01$ | $1.07 \mathrm{E}+00$ | $1.27 \mathrm{E}-02$ | $6.68 \mathrm{E}+01$ |
|  | DBBO | $6.35 \mathrm{E}+00$ | $3.01 \mathrm{E}+00$ | $6.97 \mathrm{E}+00$ | $5.95 \mathrm{E}-03$ | $9.61 \mathrm{E}+00$ |
|  | LBBO | $1.54 \mathrm{E}+01$ | $1.92 \mathrm{E}+01$ | 6.34E-01 | $3.78 \mathrm{E}-04$ | $6.63 \mathrm{E}+01$ |
|  | BBBO | $2.42 \mathrm{E}+01$ | $2.57 \mathrm{E}+01$ | $5.48 \mathrm{E}+00$ | $1.29 \mathrm{E}-02$ | $6.82 \mathrm{E}+01$ |
|  | FBBO | $2.75 \mathrm{E}+00$ | $2.31 \mathrm{E}+01$ | $3.48 \mathrm{E}+01$ | $7.87 \mathrm{E}-06$ | $7.77 \mathrm{E}+01$ |
| $f_{5}$ | GSA | $2.00 \mathrm{E}+01$ | 3.78E-04 | $2.00 \mathrm{E}+01$ | $2.00 \mathrm{E}+01$ | $2.00 \mathrm{E}+01$ |
|  | BBO | $1.89 \mathrm{E}+01$ | $4.38 \mathrm{E}+00$ | $2.00 \mathrm{E}+01$ | $3.71 \mathrm{E}-01$ | $2.01 \mathrm{E}+01$ |
|  | M1BBO | $2.00 \mathrm{E}+01$ | $5.16 \mathrm{E}-03$ | $2.00 \mathrm{E}+01$ | $2.00 \mathrm{E}+01$ | $2.00 \mathrm{E}+01$ |
|  | M2BBO | $1.87 \mathrm{E}+01$ | $4.66 \mathrm{E}+00$ | $2.00 \mathrm{E}+01$ | $6.60 \mathrm{E}-01$ | $2.00 \mathrm{E}+01$ |
|  | DBBO | $2.00 \mathrm{E}+01$ | $8.04 \mathrm{E}-03$ | $2.00 \mathrm{E}+01$ | $2.00 \mathrm{E}+01$ | $2.00 \mathrm{E}+01$ |
|  | LBBO | $1.93 \mathrm{E}+01$ | $3.55 \mathrm{E}+00$ | $2.00 \mathrm{E}+01$ | $2.98 \mathrm{E}-06$ | $2.00 \mathrm{E}+01$ |
|  | BBBO | $1.97 \mathrm{E}+01$ | $2.50 \mathrm{E}+00$ | $2.00 \mathrm{E}+01$ | $2.01 \mathrm{E}+00$ | $2.01 \mathrm{E}+01$ |
|  | FBBO | $1.00 \mathrm{E}+01$ | $1.85 \mathrm{E}-03$ | $2.00 \mathrm{E}+01$ | $1.98 \mathrm{E}-06$ | $2.00 \mathrm{E}+01$ |
| $f_{6}$ | GSA | $4.51 \mathrm{E}+00$ | $1.71 \mathrm{E}+00$ | $4.58 \mathrm{E}+00$ | $1.50 \mathrm{E}+00$ | $6.23 \mathrm{E}+00$ |
|  | BBO | $2.24 \mathrm{E}+00$ | $1.23 \mathrm{E}+00$ | $2.07 \mathrm{E}+00$ | $2.56 \mathrm{E}-01$ | $5.70 \mathrm{E}+00$ |
|  | M1BBO | $1.44 \mathrm{E}+00$ | $1.16 \mathrm{E}+00$ | $1.51 \mathrm{E}+00$ | $2.00 \mathrm{E}-02$ | $5.71 \mathrm{E}+00$ |
|  | M2BBO | $2.34 \mathrm{E}+00$ | $1.22 \mathrm{E}+00$ | $2.25 \mathrm{E}+00$ | $3.20 \mathrm{E}-01$ | $4.67 \mathrm{E}+00$ |
|  | DBBO | $1.52 \mathrm{E}+00$ | $1.19 \mathrm{E}+00$ | $1.71 \mathrm{E}+00$ | $1.52 \mathrm{E}-01$ | $5.25 \mathrm{E}+00$ |
|  | LBBO | $2.49 \mathrm{E}+00$ | $1.31 \mathrm{E}+00$ | $2.26 \mathrm{E}+00$ | $2.67 \mathrm{E}-01$ | $5.77 \mathrm{E}+00$ |
|  | BBBO | $2.87 \mathrm{E}+00$ | 1.12E +00 | $2.92 \mathrm{E}+00$ | $7.16 \mathrm{E}-01$ | $5.93 \mathrm{E}+00$ |
|  | FBBO | $1.33 \mathrm{E}+00$ | $1.46 \mathrm{E}+00$ | $4.01 \mathrm{E}+00$ | 1.62E-02 | $7.52 \mathrm{E}+00$ |

Table 2 continued

| TP | Algorithms | Mean error | SD | Med | Best | Worst |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{7}$ | GSA | $8.21 \mathrm{E}-04$ | $3.57 \mathrm{E}-03$ | $8.21 \mathrm{E}-09$ | $2.08 \mathrm{E}-09$ | $1.72 \mathrm{E}-02$ |
|  | BBO | $3.10 \mathrm{E}-01$ | $1.18 \mathrm{E}-01$ | $3.07 \mathrm{E}-01$ | $6.23 \mathrm{E}-02$ | $7.19 \mathrm{E}-01$ |
|  | M1BBO | $5.51 \mathrm{E}-02$ | $4.95 \mathrm{E}-02$ | $3.94 \mathrm{E}-02$ | $2.85 \mathrm{E}-06$ | $2.27 \mathrm{E}-01$ |
|  | M2BBO | $4.95 \mathrm{E}-02$ | $1.01 \mathrm{E}-01$ | $2.92 \mathrm{E}-01$ | $2.11 \mathrm{E}-06$ | $6.63 \mathrm{E}-01$ |
|  | DBBO | $9.12 \mathrm{E}-02$ | $5.91 \mathrm{E}-02$ | $7.80 \mathrm{E}-02$ | $1.33 \mathrm{E}-02$ | $2.72 \mathrm{E}-01$ |
|  | LBBO | $1.74 \mathrm{E}-01$ | $1.02 \mathrm{E}-01$ | $1.38 \mathrm{E}-01$ | $2.95 \mathrm{E}-02$ | $4.45 \mathrm{E}-01$ |
|  | BBBO | $3.21 \mathrm{E}-01$ | $1.34 \mathrm{E}-01$ | $2.99 \mathrm{E}-01$ | $1.14 \mathrm{E}-01$ | $6.72 \mathrm{E}-01$ |
|  | FBBO | $3.85 \mathrm{E}-04$ | $2.30 \mathrm{E}-01$ | $3.24 \mathrm{E}-01$ | $2.00 \mathrm{E}-09$ | $1.03 \mathrm{E}+00$ |
| $f_{8}$ | GSA | $3.53 \mathrm{E}+01$ | $5.47 \mathrm{E}+00$ | $3.48 \mathrm{E}+01$ | $2.59 \mathrm{E}+01$ | $4.97 \mathrm{E}+01$ |
|  | BBO | $1.79 \mathrm{E}-02$ | $1.26 \mathrm{E}-02$ | $1.51 \mathrm{E}-02$ | $1.23 \mathrm{E}-03$ | $6.42 \mathrm{E}-02$ |
|  | M1BBO | $4.20 \mathrm{E}-07$ | $2.36 \mathrm{E}-07$ | $3.93 \mathrm{E}-07$ | $8.97 \mathrm{E}-08$ | $1.12 \mathrm{E}-06$ |
|  | M2BBO | $1.94 \mathrm{E}-02$ | $1.49 \mathrm{E}-02$ | $1.48 \mathrm{E}-02$ | $3.64 \mathrm{E}-03$ | $7.19 \mathrm{E}-02$ |
|  | DBBO | $1.55 \mathrm{E}-02$ | $2.37 \mathrm{E}-02$ | $8.34 \mathrm{E}-03$ | $1.79 \mathrm{E}-05$ | $1.56 \mathrm{E}-01$ |
|  | LBBO | 1.10E-11 | 1.67E-11 | 3.98E-12 | $2.27 \mathrm{E}-13$ | $7.01 \mathrm{E}-11$ |
|  | BBBO | $1.03 \mathrm{E}-03$ | $1.88 \mathrm{E}-03$ | $3.18 \mathrm{E}-04$ | $1.58 \mathrm{E}-05$ | $1.07 \mathrm{E}-02$ |
|  | FBBO | $9.10 \mathrm{E}-08$ | $2.78 \mathrm{E}-07$ | $2.49 \mathrm{E}-07$ | $5.43 \mathrm{E}-09$ | $1.81 \mathrm{E}-04$ |
| $f_{9}$ | GSA | $3.14 \mathrm{E}+01$ | $5.58 \mathrm{E}+00$ | $3.08 \mathrm{E}+01$ | $1.79 \mathrm{E}+01$ | $4.38 \mathrm{E}+01$ |
|  | BBO | $8.03 \mathrm{E}+00$ | $2.83 \mathrm{E}+00$ | $7.98 \mathrm{E}+00$ | $2.01 \mathrm{E}+00$ | $1.49 \mathrm{E}+01$ |
|  | M1BBO | $9.40 \mathrm{E}+00$ | $3.66 \mathrm{E}+00$ | $8.95 \mathrm{E}+00$ | $2.98 \mathrm{E}+00$ | $2.29 \mathrm{E}+01$ |
|  | M2BBO | $9.46 \mathrm{E}+00$ | $3.44 \mathrm{E}+00$ | $9.96 \mathrm{E}+00$ | $3.99 \mathrm{E}+00$ | $1.60 \mathrm{E}+01$ |
|  | DBBO | $1.02 \mathrm{E}+01$ | $5.75 \mathrm{E}+00$ | $9.95 \mathrm{E}+00$ | $3.00 \mathrm{E}+00$ | $2.99 \mathrm{E}+01$ |
|  | LBBO | $8.72 \mathrm{E}+00$ | $3.48 \mathrm{E}+00$ | $8.95 \mathrm{E}+00$ | $2.98 \mathrm{E}+00$ | $1.79 \mathrm{E}+01$ |
|  | BBBO | $1.20 \mathrm{E}+01$ | $4.14 \mathrm{E}+00$ | $1.19 \mathrm{E}+01$ | $3.98 \mathrm{E}+00$ | $2.02 \mathrm{E}+01$ |
|  | FBBO | $1.81 \mathrm{E}+01$ | $6.52 \mathrm{E}+00$ | $1.79 \mathrm{E}+01$ | $4.98 \mathrm{E}+00$ | $3.18 \mathrm{E}+01$ |
| $f_{10}$ | GSA | $8.91 \mathrm{E}+02$ | $2.58 \mathrm{E}+02$ | $8.94 \mathrm{E}+02$ | $4.61 \mathrm{E}+02$ | $1.47 \mathrm{E}+03$ |
|  | BBO | $2.66 \mathrm{E}-01$ | $1.26 \mathrm{E}-01$ | $2.29 \mathrm{E}-01$ | $5.00 \mathrm{E}-02$ | $6.00 \mathrm{E}-01$ |
|  | M1BBO | $1.87 \mathrm{E}-01$ | $8.13 \mathrm{E}-02$ | $1.88 \mathrm{E}-01$ | $8.66 \mathrm{E}-03$ | $3.75 \mathrm{E}-01$ |
|  | M2BBO | $1.67 \mathrm{E}-01$ | $1.01 \mathrm{E}-01$ | $2.48 \mathrm{E}-01$ | $7.26 \mathrm{E}-02$ | $6.47 \mathrm{E}-01$ |
|  | DBBO | $3.10 \mathrm{E}-01$ | $2.93 \mathrm{E}-01$ | $2.40 \mathrm{E}-01$ | $6.43 \mathrm{E}-02$ | $2.00 \mathrm{E}+00$ |
|  | LBBO | $6.86 \mathrm{E}-02$ | 5.57E-02 | $6.25 \mathrm{E}-02$ | $1.00 \mathrm{E}-08$ | $2.50 \mathrm{E}-01$ |
|  | BBBO | $5.20 \mathrm{E}-01$ | $6.13 \mathrm{E}-01$ | $3.40 \mathrm{E}-01$ | $1.19 \mathrm{E}-01$ | $3.67 \mathrm{E}+00$ |
|  | FBBO | $5.00 \mathrm{E}-03$ | $1.85 \mathrm{E}+00$ | $5.90 \mathrm{E}-02$ | $7.27 \mathrm{E}-04$ | 1.15E-01 |
| $f_{11}$ | GSA | $1.00 \mathrm{E}+03$ | $2.36 \mathrm{E}+02$ | $1.00 \mathrm{E}+03$ | $4.69 \mathrm{E}+02$ | $1.65 \mathrm{E}+03$ |
|  | BBO | $3.24 \mathrm{E}+02$ | $1.85 \mathrm{E}+02$ | $3.16 \mathrm{E}+02$ | $1.12 \mathrm{E}+01$ | $6.67 \mathrm{E}+02$ |
|  | M1BBO | $3.26 \mathrm{E}+02$ | $2.02 \mathrm{E}+02$ | $2.59 \mathrm{E}+02$ | $1.18 \mathrm{E}+01$ | $7.90 \mathrm{E}+02$ |
|  | M2BBO | $3.18 \mathrm{E}+02$ | $1.56 \mathrm{E}+02$ | $2.78 \mathrm{E}+02$ | $1.07 \mathrm{E}+01$ | $7.24 \mathrm{E}+02$ |
|  | DBBO | $4.91 \mathrm{E}+02$ | $2.42 \mathrm{E}+02$ | $4.61 \mathrm{E}+02$ | $3.22 \mathrm{E}+01$ | $9.94 \mathrm{E}+02$ |
|  | LBBO | $4.17 \mathrm{E}+02$ | $2.25 \mathrm{E}+02$ | $3.91 \mathrm{E}+02$ | $1.86 \mathrm{E}+01$ | $1.04 \mathrm{E}+03$ |
|  | BBBO | $2.71 \mathrm{E}+02$ | $1.55 \mathrm{E}+02$ | $2.52 \mathrm{E}+02$ | $6.99 \mathrm{E}+00$ | $8.04 \mathrm{E}+02$ |
|  | FbBO | $2.18 \mathrm{E}+01$ | $1.04 \mathrm{E}+02$ | $1.82 \mathrm{E}+02$ | $1.56 \mathrm{E}+00$ | $1.36 \mathrm{E}+02$ |
| $f_{12}$ | GSA | 5.93E-04 | $2.99 \mathrm{E}-03$ | $9.00 \mathrm{E}-09$ | 6.36E-09 | $1.51 \mathrm{E}-02$ |
|  | BBO | $1.40 \mathrm{E}-01$ | $5.34 \mathrm{E}-02$ | $1.29 \mathrm{E}-01$ | $5.58 \mathrm{E}-02$ | $3.13 \mathrm{E}-01$ |
|  | M1bBo | $6.35 \mathrm{E}-02$ | $5.96 \mathrm{E}-02$ | $4.67 \mathrm{E}-02$ | $3.39 \mathrm{E}-03$ | $3.26 \mathrm{E}-01$ |
|  | M2BBO | $1.32 \mathrm{E}-01$ | $6.04 \mathrm{E}-02$ | $1.23 \mathrm{E}-01$ | $4.28 \mathrm{E}-02$ | $3.45 \mathrm{E}-01$ |
|  | DBBO | $1.25 \mathrm{E}-01$ | $6.20 \mathrm{E}-02$ | $1.17 \mathrm{E}-01$ | $1.38 \mathrm{E}-02$ | $2.84 \mathrm{E}-01$ |

Table 2 continued


Table 2 continued

| TP | Algorithms | Mean error | SD | Med | Best | Worst |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{19}$ | M2BBO | $1.24 \mathrm{E}+04$ | $1.10 \mathrm{E}+04$ | $8.76 \mathrm{E}+03$ | $1.53 \mathrm{E}+02$ | $5.28 \mathrm{E}+04$ |
|  | DBBO | $1.01 \mathrm{E}+04$ | $1.01 \mathrm{E}+04$ | $5.87 \mathrm{E}+03$ | $6.92 \mathrm{E}+00$ | $3.38 \mathrm{E}+04$ |
|  | LBBO | $9.93 \mathrm{E}+03$ | $7.22 \mathrm{E}+03$ | $9.02 \mathrm{E}+03$ | $1.67 \mathrm{E}+02$ | $3.00 \mathrm{E}+04$ |
|  | BBBO | $8.21 \mathrm{E}+03$ | $3.44 \mathrm{E}+03$ | $8.18 \mathrm{E}+03$ | $7.41 \mathrm{E}+02$ | $1.82 \mathrm{E}+04$ |
|  | FBBO | $5.55 \mathrm{E}+03$ | $\mathbf{1 . 5 8 E}+03$ | $4.02 \mathrm{E}+03$ | $4.14 \mathrm{E}+00$ | 1.05E +04 |
|  | GSA | $3.84 \mathrm{E}+00$ | $9.12 \mathrm{E}-01$ | $3.35 \mathrm{E}+00$ | $2.50 \mathrm{E}+00$ | $5.19 \mathrm{E}+00$ |
|  | BBO | $9.60 \mathrm{E}-01$ | $7.23 \mathrm{E}-01$ | $7.46 \mathrm{E}-01$ | $1.31 \mathrm{E}-01$ | $4.09 \mathrm{E}+00$ |
|  | M1BBO | $6.78 \mathrm{E}-01$ | $6.89 \mathrm{E}-01$ | $4.75 \mathrm{E}-01$ | $3.80 \mathrm{E}-02$ | $4.24 \mathrm{E}+00$ |
|  | M2BBO | $7.91 \mathrm{E}-01$ | $4.10 \mathrm{E}-01$ | $7.18 \mathrm{E}-01$ | $2.62 \mathrm{E}-01$ | $2.16 \mathrm{E}+00$ |
|  | DBBO | $1.30 \mathrm{E}+00$ | $7.06 \mathrm{E}-01$ | $1.19 \mathrm{E}+00$ | $1.75 \mathrm{E}-01$ | $4.23 \mathrm{E}+00$ |
| $f_{20}$ | LBBO | $7.31 \mathrm{E}-01$ | $4.25 \mathrm{E}-01$ | $7.34 \mathrm{E}-01$ | $7.69 \mathrm{E}-02$ | $1.97 \mathrm{E}+00$ |
|  | BBBO | $5.88 \mathrm{E}-01$ | $4.07 \mathrm{E}-01$ | $4.46 \mathrm{E}-01$ | $2.10 \mathrm{E}-01$ | $2.10 \mathrm{E}+00$ |
|  | FBBO | $3.40 \mathrm{E}-01$ | $2.85 \mathrm{E}-01$ | 3.52E-01 | 3.55E-02 | 1.68E+00 |
|  | GSA | $1.67 \mathrm{E}+04$ | $1.72 \mathrm{E}+04$ | $1.09 \mathrm{E}+04$ | $3.35 \mathrm{E}+03$ | $8.26 \mathrm{E}+04$ |
|  | BBO | $9.77 \mathrm{E}+03$ | $9.44 \mathrm{E}+03$ | $6.21 \mathrm{E}+03$ | $2.07 \mathrm{E}+01$ | $3.32 \mathrm{E}+04$ |
|  | M1BBO | $6.28 \mathrm{E}+03$ | $7.44 \mathrm{E}+03$ | $\mathbf{2 . 7 9 E}+03$ | $4.69 \mathrm{E}+00$ | $2.77 \mathrm{E}+04$ |
|  | M2BBO | $1.01 \mathrm{E}+04$ | $9.62 \mathrm{E}+03$ | $6.64 \mathrm{E}+03$ | $5.72 \mathrm{E}+01$ | $3.22 \mathrm{E}+04$ |
| $f_{21}$ | DBBO | $7.91 \mathrm{E}+03$ | $8.96 \mathrm{E}+03$ | $4.43 \mathrm{E}+03$ | $1.56 \mathrm{E}+00$ | $3.09 \mathrm{E}+04$ |
|  | LBBO | $5.77 \mathrm{E}+03$ | $5.99 \mathrm{E}+03$ | $3.47 \mathrm{E}+03$ | $4.58 \mathrm{E}+00$ | $2.76 \mathrm{E}+04$ |
|  | BBBO | $5.17 \mathrm{E}+03$ | $3.40 \mathrm{E}+03$ | $4.12 \mathrm{E}+03$ | $9.22 \mathrm{E}+02$ | $1.94 \mathrm{E}+04$ |
|  | FBBO | $6.32 \mathrm{E}+03$ | $6.73 \mathrm{E}+03$ | $4.11 \mathrm{E}+03$ | $1.25 \mathrm{E}+01$ | $2.72 \mathrm{E}+04$ |
|  | GSA | $1.79 \mathrm{E}+05$ | $2.98 \mathrm{E}+05$ | $1.40 \mathrm{E}+05$ | $1.11 \mathrm{E}+04$ | $5.82 \mathrm{E}+05$ |
|  | BBO | $5.56 \mathrm{E}+05$ | $7.20 \mathrm{E}+05$ | $3.82 \mathrm{E}+05$ | $9.04 \mathrm{E}+03$ | $4.28 \mathrm{E}+06$ |
|  | M1BBO | $7.23 \mathrm{E}+04$ | $1.25 \mathrm{E}+05$ | $2.19 \mathrm{E}+04$ | $1.55 \mathrm{E}+02$ | $5.39 \mathrm{E}+05$ |
|  | M2BBO | $5.05 \mathrm{E}+05$ | $6.76 \mathrm{E}+05$ | $2.08 \mathrm{E}+05$ | $2.10 \mathrm{E}+03$ | $3.49 \mathrm{E}+06$ |
|  | DBBO | $7.89 \mathrm{E}+03$ | $1.07 \mathrm{E}+04$ | $4.46 \mathrm{E}+03$ | $5.70 \mathrm{E}+01$ | $6.00 \mathrm{E}+04$ |
| $f_{22}$ | LBBO | $3.66 \mathrm{E}+05$ | $5.94 \mathrm{E}+05$ | $6.46 \mathrm{E}+04$ | $1.09 \mathrm{E}+03$ | $3.14 \mathrm{E}+06$ |
|  | BBBO | $2.97 \mathrm{E}+05$ | $3.60 \mathrm{E}+05$ | $1.51 \mathrm{E}+05$ | $8.82 \mathrm{E}+01$ | $1.37 \mathrm{E}+06$ |
|  | FBBO | $6.89 \mathrm{E}+03$ | $6.35 \mathrm{E}+03$ | $5.47 \mathrm{E}+03$ | $1.55 \mathrm{E}+01$ | $2.65 \mathrm{E}+04$ |
|  | GSA | $2.41 \mathrm{E}+02$ | $1.17 \mathrm{E}+02$ | $1.73 \mathrm{E}+02$ | $1.45 \mathrm{E}+02$ | $3.86 \mathrm{E}+02$ |
|  | BBO | $2.16 \mathrm{E}+01$ | $4.55 \mathrm{E}+01$ | $2.10 \mathrm{E}+00$ | $5.50 \mathrm{E}-01$ | $1.49 \mathrm{E}+02$ |
|  | M1BBO | $2.03 \mathrm{E}+01$ | $4.86 \mathrm{E}+01$ | $5.02 \mathrm{E}-01$ | 2.02E-02 | $1.90 \mathrm{E}+02$ |
|  | M2BBO | $3.37 \mathrm{E}+01$ | $6.21 \mathrm{E}+01$ | $2.60 \mathrm{E}+00$ | $3.59 \mathrm{E}-01$ | $2.22 \mathrm{E}+02$ |
| $f_{23}$ | DBBO | $2.92 \mathrm{E}+01$ | $4.80 \mathrm{E}+01$ | $3.66 \mathrm{E}+00$ | $1.03 \mathrm{E}-01$ | $1.47 \mathrm{E}+02$ |
|  | LBBO | $6.09 \mathrm{E}+01$ | $7.00 \mathrm{E}+01$ | $2.48 \mathrm{E}+00$ | $6.46 \mathrm{E}-02$ | $1.90 \mathrm{E}+02$ |
|  | BBBO | $6.01 \mathrm{E}+01$ | $6.35 \mathrm{E}+01$ | $1.08 \mathrm{E}+00$ | $3.62 \mathrm{E}-01$ | $1.44 \mathrm{E}+02$ |
|  | FBBO | $2.03 \mathrm{E}+01$ | $5.50 \mathrm{E}+01$ | $4.60 \mathrm{E}-01$ | $2.12 \mathrm{E}-02$ | $1.63 \mathrm{E}+02$ |
|  | GSA | $2.71 \mathrm{E}+02$ | $9.59 \mathrm{E}+01$ | $3.29 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ |
|  | BBO | $3.29 \mathrm{E}+02$ | $2.06 \mathrm{E}-02$ | $3.29 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ | $3.30 \mathrm{E}+02$ |
|  | M1BBO | $3.29 \mathrm{E}+02$ | $1.63 \mathrm{E}-07$ | $3.29 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ |
|  | M2BBO | $3.29 \mathrm{E}+02$ | $2.18 \mathrm{E}-02$ | $3.29 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ | $3.30 \mathrm{E}+02$ |
|  | DBBO | $3.29 \mathrm{E}+02$ | $2.05 \mathrm{E}-02$ | $3.29 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ | $3.30 \mathrm{E}+02$ |
|  | LBBO | $3.29 \mathrm{E}+02$ | $6.29 \mathrm{E}-07$ | $3.29 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ |
|  | BBBO | $3.29 \mathrm{E}+02$ | $2.32 \mathrm{E}-04$ | $3.29 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ |

Table 2 continued


Table 2 continued

Table 3 Comparison of FBBO, GSA, and other variant of BBO for 30-dimensional CEC 2014 benchmark problems based on the obtained average, standard deviation, median, best, worst error value

| TP | Algorithms | Mean error | SD | Med | Best | Worst |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f_{29}$ | GSA | $1.10 \mathrm{E}+06$ | $5.02 \mathrm{E}+06$ | $\mathbf{2 . 0 0 E}+\mathbf{0 2}$ | $2.00 \mathrm{E}+02$ | $2.60 \mathrm{E}+07$ |
|  | BBO | $5.03 \mathrm{E}+02$ | $1.83 \mathrm{E}+02$ | $4.91 \mathrm{E}+02$ | $2.72 \mathrm{E}+02$ | $1.30 \mathrm{E}+03$ |
|  | M1BBO | $2.06 \mathrm{E}+02$ | $\mathbf{2 . 8 5 E}+\mathbf{0 0}$ | $2.06 \mathrm{E}+02$ | $2.03 \mathrm{E}+02$ | $2.22 \mathrm{E}+02$ |
|  | M2BBO | $5.09 \mathrm{E}+02$ | $2.05 \mathrm{E}+02$ | $4.73 \mathrm{E}+02$ | $2.39 \mathrm{E}+02$ | $1.22 \mathrm{E}+03$ |
|  | DBBO | $\mathbf{2 . 0 6 E}+\mathbf{0 2}$ | $3.82 \mathrm{E}+00$ | $2.05 \mathrm{E}+02$ | $2.02 \mathrm{E}+02$ | $\mathbf{2 . 2 0 E}+\mathbf{0 2}$ |
|  | LBBO | $3.60 \mathrm{E}+02$ | $1.26 \mathrm{E}+02$ | $3.11 \mathrm{E}+02$ | $1.95 \mathrm{E}+02$ | $8.47 \mathrm{E}+02$ |
|  | BBBO | $3.57 \mathrm{E}+04$ | $2.50 \mathrm{E}+05$ | $3.01 \mathrm{E}+02$ | $\mathbf{1 . 8 3 E}+\mathbf{0 2}$ | $1.80 \mathrm{E}+06$ |
|  | FBBO | $4.86 \mathrm{E}+02$ | $2.11 \mathrm{E}+02$ | $4.24 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ | $1.16 \mathrm{E}+03$ |
| $f_{30}$ | GSA | $2.85 \mathrm{E}+03$ | $5.48 \mathrm{E}+02$ | $2.77 \mathrm{E}+03$ | $2.06 \mathrm{E}+03$ | $4.12 \mathrm{E}+03$ |
|  | BBO | $8.55 \mathrm{E}+02$ | $3.93 \mathrm{E}+02$ | $7.99 \mathrm{E}+02$ | $4.89 \mathrm{E}+02$ | $3.29 \mathrm{E}+03$ |
|  | M1BBO | $3.95 \mathrm{E}+02$ | $1.09 \mathrm{E}+02$ | $4.01 \mathrm{E}+02$ | $2.34 \mathrm{E}+02$ | $6.63 \mathrm{E}+02$ |
|  | M2BBO | $7.94 \mathrm{E}+02$ | $2.10 \mathrm{E}+02$ | $7.53 \mathrm{E}+02$ | $4.99 \mathrm{E}+02$ | $1.46 \mathrm{E}+03$ |
|  | DBBO | $\mathbf{3 . 3 5 E}+\mathbf{0 2}$ | $\mathbf{8 . 1 5 E}+\mathbf{0 1}$ | $\mathbf{3 . 2 7 E}+\mathbf{0 2}$ | $2.33 \mathrm{E}+02$ | $\mathbf{6 . 1 1 E}+\mathbf{0 2}$ |
|  | LBBO | $1.12 \mathrm{E}+03$ | $2.57 \mathrm{E}+02$ | $1.13 \mathrm{E}+03$ | $5.19 \mathrm{E}+02$ | $1.81 \mathrm{E}+03$ |
|  | BBBO | $1.25 \mathrm{E}+03$ | $2.82 \mathrm{E}+02$ | $1.25 \mathrm{E}+03$ | $4.22 \mathrm{E}+02$ | $1.96 \mathrm{E}+03$ |
|  | FBBO | $3.58 \mathrm{E}+02$ | $3.11 \mathrm{E}+02$ | $6.65 \mathrm{E}+02$ | $\mathbf{2 . 0 0 E}+\mathbf{0 2}$ | $1.67 \mathrm{E}+03$ |

The better results are highlighted by bold

| TP | Algorithms | Mean error | SD | Med | Best | Worst |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $f_{1}$ | GSA | $5.30 \mathrm{E}+06$ | $6.86 \mathrm{E}+06$ | $2.16 \mathrm{E}+06$ | $1.10 \mathrm{E}+06$ | $2.46 \mathrm{E}+07$ |
|  | BBO | $1.54 \mathrm{E}+07$ | $1.60 \mathrm{E}+07$ | $1.38 \mathrm{E}+07$ | $7.06 \mathrm{E}+05$ | $4.53 \mathrm{E}+07$ |
|  | M1BBO | $2.07 \mathrm{E}+06$ | $8.21 \mathrm{E}+05$ | $1.95 \mathrm{E}+06$ | $8.46 \mathrm{E}+05$ | $4.86 \mathrm{E}+06$ |
|  | M2BBO | $1.24 \mathrm{E}+07$ | $9.00 \mathrm{E}+06$ | $8.90 \mathrm{E}+06$ | $1.71 \mathrm{E}+06$ | $3.82 \mathrm{E}+07$ |
|  | DBBO | $4.35 \mathrm{E}+06$ | $2.46 \mathrm{E}+06$ | $4.04 \mathrm{E}+06$ | $1.03 \mathrm{E}+06$ | $1.11 \mathrm{E}+07$ |
|  | LBBO | $2.40 \mathrm{E}+06$ | $1.05 \mathrm{E}+06$ | $2.13 \mathrm{E}+06$ | $8.05 \mathrm{E}+05$ | $6.05 \mathrm{E}+06$ |
|  | BBBO | $7.72 \mathrm{E}+06$ | $4.08 \mathrm{E}+06$ | $6.88 \mathrm{E}+06$ | $3.56 \mathrm{E}+06$ | $2.86 \mathrm{E}+07$ |
|  | FBBO | $\mathbf{1 . 1 4 E}+\mathbf{0 5}$ | $\mathbf{8 . 1 4 E}+\mathbf{0 5}$ | $\mathbf{1 . 3 3 E}+\mathbf{0 6}$ | $\mathbf{2 . 0 3 E + 0 4}$ | $\mathbf{2 . 3 0 E}+\mathbf{0 6}$ |
| $f_{2}$ | GSA | $8.51 \mathrm{E}+03$ | $4.40 \mathrm{E}+03$ | $7.80 \mathrm{E}+03$ | $2.94 \mathrm{E}+03$ | $1.79 \mathrm{E}+04$ |
|  | BBO | $2.25 \mathrm{E}+05$ | $1.10 \mathrm{E}+05$ | $2.14 \mathrm{E}+05$ | $8.78 \mathrm{E}+04$ | $6.02 \mathrm{E}+05$ |
|  | M1BBO | $1.38 \mathrm{E}+04$ | $1.36 \mathrm{E}+04$ | $1.00 \mathrm{E}+04$ | $\mathbf{2 . 6 9 E}+\mathbf{0 2}$ | $5.50 \mathrm{E}+04$ |
|  | M2BBO | $2.19 \mathrm{E}+05$ | $9.36 \mathrm{E}+04$ | $1.98 \mathrm{E}+05$ | $7.78 \mathrm{E}+04$ | $5.30 \mathrm{E}+05$ |
|  | DBBO | $2.59 \mathrm{E}+04$ | $2.30 \mathrm{E}+04$ | $1.53 \mathrm{E}+04$ | $1.14 \mathrm{E}+03$ | $9.23 \mathrm{E}+04$ |
|  | LBBO | $1.11 \mathrm{E}+04$ | $4.71 \mathrm{E}+03$ | $1.05 \mathrm{E}+04$ | $2.34 \mathrm{E}+03$ | $2.23 \mathrm{E}+04$ |
|  | BBBO | $6.37 \mathrm{E}+04$ | $4.96 \mathrm{E}+04$ | $4.28 \mathrm{E}+04$ | $2.01 \mathrm{E}+04$ | $2.59 \mathrm{E}+05$ |
|  | FBBO | $\mathbf{3 . 7 7 E + \mathbf { 0 3 }}$ | $\mathbf{3 . 9 6 E + 0 3}$ | $\mathbf{6 . 9 2 E + 0 3}$ | $5.58 \mathrm{E}+02$ | $\mathbf{1 . 7 6 E + 0 4}$ |
| $f_{3}$ | GSA | $1.29 \mathrm{E}+04$ | $8.69 \mathrm{E}+03$ | $1.22 \mathrm{E}+04$ | $3.90 \mathrm{E}+03$ | $3.03 \mathrm{E}+04$ |
|  | BBO | $1.56 \mathrm{E}+04$ | $2.06 \mathrm{E}+04$ | $1.28 \mathrm{E}+04$ | $3.83 \mathrm{E}+02$ | $6.41 \mathrm{E}+04$ |
|  | M1BBO | $1.25 \mathrm{E}+04$ | $1.48 \mathrm{E}+04$ | $7.13 \mathrm{E}+03$ | $9.23 \mathrm{E}+01$ | $6.35 \mathrm{E}+04$ |
|  | M2BBO | $1.84 \mathrm{E}+04$ | $1.61 \mathrm{E}+04$ | $1.32 \mathrm{E}+04$ | $\mathbf{1 . 1 5 E}+\mathbf{0 1}$ | $8.26 \mathrm{E}+04$ |
|  | DBBO | $2.23 \mathrm{E}+04$ | $2.35 \mathrm{E}+04$ | $1.82 \mathrm{E}+04$ | $5.09 \mathrm{E}+02$ | $1.45 \mathrm{E}+05$ |
|  | LBBO | $1.16 \mathrm{E}+04$ | $1.19 \mathrm{E}+04$ | $8.27 \mathrm{E}+03$ | $3.45 \mathrm{E}+02$ | $6.53 \mathrm{E}+04$ |
|  | BBBO | $7.13 \mathrm{E}+03$ | $\mathbf{4 . 8 9 E + \mathbf { 0 3 }}$ | $5.41 \mathrm{E}+03$ | $1.37 \mathrm{E}+03$ | $2.38 \mathrm{E}+04$ |

Table 3 continued


Table 3 continued

| TP | Algorithms | Mean error | SD | Med | Best | Worst |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{9}$ | GSA | $1.56 \mathrm{E}+02$ | $1.53 \mathrm{E}+01$ | $1.54 \mathrm{E}+02$ | $1.19 \mathrm{E}+02$ | $1.97 \mathrm{E}+02$ |
|  | BBO | $5.06 \mathrm{E}+01$ | $1.73 \mathrm{E}+01$ | $4.68 \mathrm{E}+01$ | $2.70 \mathrm{E}+01$ | $8.46 \mathrm{E}+01$ |
|  | M1BBO | $5.54 \mathrm{E}+01$ | $1.40 \mathrm{E}+01$ | $5.37 \mathrm{E}+01$ | 1.89E+01 | 8.46E +01 |
|  | M2BBO | $5.32 \mathrm{E}+01$ | 1.15E +01 | $5.28 \mathrm{E}+01$ | $3.09 \mathrm{E}+01$ | $8.67 \mathrm{E}+01$ |
|  | DBBO | $5.06 \mathrm{E}+01$ | $1.75 \mathrm{E}+01$ | $7.76 \mathrm{E}+01$ | $4.18 \mathrm{E}+01$ | $1.09 \mathrm{E}+02$ |
|  | LBBO | $5.60 \mathrm{E}+01$ | $1.37 \mathrm{E}+01$ | $5.67 \mathrm{E}+01$ | $2.39 \mathrm{E}+01$ | $8.46 \mathrm{E}+01$ |
|  | BBBO | $8.38 \mathrm{E}+01$ | $1.59 \mathrm{E}+01$ | $8.06 \mathrm{E}+01$ | $5.77 \mathrm{E}+01$ | $1.28 \mathrm{E}+02$ |
|  | FBBO | $5.16 \mathrm{E}+01$ | $2.37 \mathrm{E}+01$ | $1.43 \mathrm{E}+02$ | $2.85 \mathrm{E}+01$ | $1.87 \mathrm{E}+02$ |
| $f_{10}$ | GSA | $3.28 \mathrm{E}+03$ | $7.79 \mathrm{E}+02$ | $3.30 \mathrm{E}+03$ | $2.51 \mathrm{E}+03$ | $4.41 \mathrm{E}+03$ |
|  | BBO | $4.58 \mathrm{E}-01$ | 1.03E-01 | $4.44 \mathrm{E}-01$ | $2.95 \mathrm{E}-01$ | $7.32 \mathrm{E}-01$ |
|  | M1BBO | $3.33 \mathrm{E}+00$ | $2.14 \mathrm{E}+00$ | $2.60 \mathrm{E}+00$ | $2.79 \mathrm{E}-01$ | $9.57 \mathrm{E}+00$ |
|  | M2BBO | $4.49 \mathrm{E}-01$ | $1.12 \mathrm{E}-01$ | $4.49 \mathrm{E}-01$ | $3.17 \mathrm{E}-01$ | $8.22 \mathrm{E}-01$ |
|  | DBBO | $1.26 \mathrm{E}+00$ | $8.90 \mathrm{E}-01$ | $1.05 \mathrm{E}+00$ | $3.46 \mathrm{E}-01$ | $4.94 \mathrm{E}+00$ |
|  | LBBO | $9.37 \mathrm{E}-01$ | $8.23 \mathrm{E}-01$ | 3.98E-01 | $1.46 \mathrm{E}-01$ | $3.51 \mathrm{E}+00$ |
|  | BBBO | $2.16 \mathrm{E}+00$ | $1.63 \mathrm{E}+00$ | $1.69 \mathrm{E}+00$ | $2.63 \mathrm{E}-01$ | $9.09 \mathrm{E}+00$ |
|  | FBBO | 2.82E-01 | $1.03 \mathrm{E}-01$ | $2.71 \mathrm{E}+00$ | 1.03E-01 | $6.07 \mathrm{E}-01$ |
| $f_{11}$ | GSA | $3.93 \mathrm{E}+03$ | $7.44 \mathrm{E}+02$ | $3.94 \mathrm{E}+03$ | $3.03 \mathrm{E}+03$ | $5.28 \mathrm{E}+03$ |
|  | BBO | $1.91 \mathrm{E}+03$ | $9.46 \mathrm{E}+02$ | $1.99 \mathrm{E}+03$ | $7.92 \mathrm{E}+02$ | $2.69 \mathrm{E}+03$ |
|  | M1BBO | $1.81 \mathrm{E}+03$ | $4.73 \mathrm{E}+02$ | $2.15 \mathrm{E}+03$ | $1.89 \mathrm{E}+02$ | $3.11 \mathrm{E}+03$ |
|  | M2BBO | $1.90 \mathrm{E}+03$ | $4.07 \mathrm{E}+02$ | $2.02 \mathrm{E}+03$ | $1.77 \mathrm{E}+02$ | $3.16 \mathrm{E}+03$ |
|  | DBBO | $1.85 \mathrm{E}+03$ | $5.31 \mathrm{E}+02$ | $2.41 \mathrm{E}+03$ | $1.39 \mathrm{E}+02$ | $3.65 \mathrm{E}+03$ |
|  | LBBO | $2.35 \mathrm{E}+03$ | $3.63 \mathrm{E}+02$ | $2.35 \mathrm{E}+03$ | $1.57 \mathrm{E}+03$ | $3.03 \mathrm{E}+03$ |
|  | BBBO | $2.46 \mathrm{E}+03$ | $4.62 \mathrm{E}+02$ | $2.47 \mathrm{E}+03$ | $1.42 \mathrm{E}+03$ | $3.53 \mathrm{E}+03$ |
|  | FBBO | $1.71 \mathrm{E}+03$ | $5.52 \mathrm{E}+02$ | $1.80 \mathrm{E}+03$ | $1.00 \mathrm{E}+02$ | $2.53 \mathrm{E}+03$ |
| $f_{12}$ | GSA | $5.41 \mathrm{E}-04$ | $9.87 \mathrm{E}-04$ | $2.25 \mathrm{E}-04$ | $1.64 \mathrm{E}-08$ | $4.99 \mathrm{E}-03$ |
|  | BBO | $1.34 \mathrm{E}-01$ | $7.84 \mathrm{E}-02$ | $1.38 \mathrm{E}-01$ | $7.41 \mathrm{E}-02$ | $2.24 \mathrm{E}-01$ |
|  | M1BBO | $1.32 \mathrm{E}-01$ | $5.77 \mathrm{E}-02$ | $1.64 \mathrm{E}-01$ | $7.66 \mathrm{E}-02$ | $3.07 \mathrm{E}-01$ |
|  | M2BBO | $1.33 \mathrm{E}-01$ | $3.60 \mathrm{E}-02$ | $1.32 \mathrm{E}-01$ | $7.24 \mathrm{E}-02$ | $2.30 \mathrm{E}-01$ |
|  | DBBO | $1.31 \mathrm{E}-01$ | $4.02 \mathrm{E}-02$ | $1.33 \mathrm{E}-01$ | $6.75 \mathrm{E}-02$ | $2.43 \mathrm{E}-01$ |
|  | LBBO | $1.47 \mathrm{E}-01$ | $5.73 \mathrm{E}-02$ | $1.33 \mathrm{E}-01$ | $4.46 \mathrm{E}-02$ | $2.99 \mathrm{E}-01$ |
|  | BBBO | $1.85 \mathrm{E}-01$ | $4.95 \mathrm{E}-02$ | $1.77 \mathrm{E}-01$ | $1.03 \mathrm{E}-01$ | $3.12 \mathrm{E}-01$ |
|  | FBBO | $1.08 \mathrm{E}-01$ | $7.38 \mathrm{E}-02$ | $2.07 \mathrm{E}-01$ | $9.10 \mathrm{E}-02$ | $4.32 \mathrm{E}-01$ |
| $f_{13}$ | GSA | $2.09 \mathrm{E}-01$ | $4.55 \mathrm{E}-02$ | $2.06 \mathrm{E}-01$ | $1.37 \mathrm{E}-01$ | $3.25 \mathrm{E}-01$ |
|  | BBO | $3.46 \mathrm{E}-01$ | $9.18 \mathrm{E}-02$ | $3.35 \mathrm{E}-01$ | $2.26 \mathrm{E}-01$ | $6.41 \mathrm{E}-01$ |
|  | M1BBO | $2.66 \mathrm{E}-01$ | $5.53 \mathrm{E}-02$ | $2.63 \mathrm{E}-01$ | $1.56 \mathrm{E}-01$ | $3.87 \mathrm{E}-01$ |
|  | M2BBO | $3.42 \mathrm{E}-01$ | $6.01 \mathrm{E}-02$ | $3.32 \mathrm{E}-01$ | $2.43 \mathrm{E}-01$ | $4.89 \mathrm{E}-01$ |
|  | DBBO | $2.66 \mathrm{E}-01$ | $6.86 \mathrm{E}-02$ | $2.69 \mathrm{E}-01$ | $1.39 \mathrm{E}-01$ | $4.31 \mathrm{E}-01$ |
|  | LBBO | $3.05 \mathrm{E}-01$ | $6.54 \mathrm{E}-02$ | $3.00 \mathrm{E}-01$ | $1.49 \mathrm{E}-01$ | $5.16 \mathrm{E}-01$ |
|  | BBBO | $2.97 \mathrm{E}-01$ | $4.21 \mathrm{E}-02$ | $3.01 \mathrm{E}-01$ | $2.04 \mathrm{E}-01$ | $4.10 \mathrm{E}-01$ |
|  | FBBO | $1.75 \mathrm{E}-01$ | $9.97 \mathrm{E}-02$ | $2.05 \mathrm{E}-01$ | 1.01E-01 | 3.21E-01 |
| $f_{14}$ | GSA | $3.07 \mathrm{E}-01$ | $7.51 \mathrm{E}-02$ | $3.05 \mathrm{E}-01$ | $1.67 \mathrm{E}-01$ | $4.16 \mathrm{E}-01$ |
|  | BBO | $3.91 \mathrm{E}-01$ | $2.77 \mathrm{E}-01$ | $2.92 \mathrm{E}-01$ | $1.96 \mathrm{E}-01$ | $8.03 \mathrm{E}-01$ |
|  | M1BBO | $2.87 \mathrm{E}-01$ | $1.00 \mathrm{E}-01$ | $2.63 \mathrm{E}-01$ | $1.60 \mathrm{E}-01$ | $6.68 \mathrm{E}-01$ |
|  | M2BBO | $3.42 \mathrm{E}-01$ | $1.83 \mathrm{E}-01$ | $2.68 \mathrm{E}-01$ | $1.43 \mathrm{E}-01$ | $8.07 \mathrm{E}-01$ |
|  | DBBO | $3.90 \mathrm{E}-01$ | $1.94 \mathrm{E}-01$ | $2.95 \mathrm{E}-01$ | $1.35 \mathrm{E}-01$ | $9.09 \mathrm{E}-01$ |
|  | LBBO | $2.18 \mathrm{E}-01$ | $2.97 \mathrm{E}-02$ | $2.17 \mathrm{E}-01$ | $1.61 \mathrm{E}-01$ | $2.75 \mathrm{E}-01$ |

Table 3 continued


Table 3 continued

| TP | Algorithms | Mean error | SD | Med | Best | Worst |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{21}$ | DBBO | $1.78 \mathrm{E}+04$ | $1.49 \mathrm{E}+04$ | $1.39 \mathrm{E}+04$ | $1.53 \mathrm{E}+03$ | $6.98 \mathrm{E}+04$ |
|  | LBBO | $2.67 \mathrm{E}+04$ | $1.29 \mathrm{E}+04$ | $2.43 \mathrm{E}+04$ | $4.94 \mathrm{E}+03$ | $6.22 \mathrm{E}+04$ |
|  | BBBO | $1.60 \mathrm{E}+04$ | $5.85 \mathrm{E}+03$ | $1.54 \mathrm{E}+04$ | $9.02 \mathrm{E}+02$ | $3.17 \mathrm{E}+04$ |
|  | FBBO | $4.10 \mathrm{E}+03$ | $4.46 \mathrm{E}+03$ | $2.48 \mathrm{E}+03$ | $2.88 \mathrm{E}+02$ | $2.84 \mathrm{E}+04$ |
|  | GSA | $1.37 \mathrm{E}+05$ | $7.45 \mathrm{E}+04$ | 1.25E +05 | $4.55 \mathrm{E}+04$ | $3.05 \mathrm{E}+05$ |
|  | BBO | $9.30 \mathrm{E}+05$ | $1.22 \mathrm{E}+06$ | $7.09 \mathrm{E}+05$ | $5.12 \mathrm{E}+04$ | $4.05 \mathrm{E}+06$ |
|  | M1BBO | $3.31 \mathrm{E}+05$ | $2.26 \mathrm{E}+05$ | $2.94 \mathrm{E}+05$ | $3.46 \mathrm{E}+04$ | $8.82 \mathrm{E}+05$ |
|  | M2BBO | $8.11 \mathrm{E}+05$ | $7.56 \mathrm{E}+05$ | $5.21 \mathrm{E}+05$ | $5.56 \mathrm{E}+04$ | $3.11 \mathrm{E}+06$ |
|  | DBBO | $8.31 \mathrm{E}+05$ | $1.16 \mathrm{E}+06$ | $8.26 \mathrm{E}+05$ | $4.39 \mathrm{E}+04$ | $4.98 \mathrm{E}+06$ |
| $f_{22}$ | LBBO | $2.81 \mathrm{E}+05$ | $2.32 \mathrm{E}+05$ | $2.12 \mathrm{E}+05$ | $1.29 \mathrm{E}+04$ | $9.03 \mathrm{E}+05$ |
|  | BBBO | $2.07 \mathrm{E}+05$ | $1.30 \mathrm{E}+05$ | $1.91 \mathrm{E}+05$ | $5.53 \mathrm{E}+04$ | $8.47 \mathrm{E}+05$ |
|  | FBBO | $1.39 \mathrm{E}+05$ | $3.39 \mathrm{E}+05$ | $3.76 \mathrm{E}+05$ | 1.21E+04 | $1.45 \mathrm{E}+06$ |
|  | GSA | $9.34 \mathrm{E}+02$ | $3.05 \mathrm{E}+02$ | $9.51 \mathrm{E}+02$ | $5.23 \mathrm{E}+02$ | $1.37 \mathrm{E}+03$ |
|  | BBO | $5.11 \mathrm{E}+02$ | $3.20 \mathrm{E}+02$ | $5.39 \mathrm{E}+02$ | $4.74 \mathrm{E}+01$ | $8.87 \mathrm{E}+02$ |
|  | M1BBO | $4.49 \mathrm{E}+02$ | $1.66 \mathrm{E}+02$ | $4.84 \mathrm{E}+02$ | $2.21 \mathrm{E}+01$ | $8.34 \mathrm{E}+02$ |
|  | M2BBO | $5.01 \mathrm{E}+02$ | $2.40 \mathrm{E}+02$ | $5.85 \mathrm{E}+02$ | $4.63 \mathrm{E}+01$ | $1.09 \mathrm{E}+03$ |
| $f_{23}$ | DBBO | $5.06 \mathrm{E}+02$ | $2.10 \mathrm{E}+02$ | $4.96 \mathrm{E}+02$ | $3.46 \mathrm{E}+01$ | $9.58 \mathrm{E}+02$ |
|  | LBBO | $5.60 \mathrm{E}+02$ | $1.93 \mathrm{E}+02$ | $5.34 \mathrm{E}+02$ | $1.81 \mathrm{E}+02$ | $9.86 \mathrm{E}+02$ |
|  | BBBO | $6.14 \mathrm{E}+02$ | $1.88 \mathrm{E}+02$ | $6.39 \mathrm{E}+02$ | $1.74 \mathrm{E}+02$ | $9.82 \mathrm{E}+02$ |
|  | FBBO | 3.65E+02 | $1.54 \mathrm{E}+02$ | $4.67 \mathrm{E}+02$ | $2.06 \mathrm{E}+01$ | $7.30 \mathrm{E}+02$ |
|  | GSA | $2.72 \mathrm{E}+02$ | $8.10 \mathrm{E}+01$ | $3.19 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ | $3.29 \mathrm{E}+02$ |
|  | BBO | $3.16 \mathrm{E}+02$ | $8.92 \mathrm{E}-01$ | $3.16 \mathrm{E}+02$ | $3.15 \mathrm{E}+02$ | $3.20 \mathrm{E}+02$ |
|  | M1BBO | $3.14 \mathrm{E}+02$ | $2.70 \mathrm{E}-02$ | $3.14 \mathrm{E}+02$ | $3.14 \mathrm{E}+02$ | $3.14 \mathrm{E}+02$ |
|  | M2BBO | $3.16 \mathrm{E}+02$ | $8.64 \mathrm{E}-01$ | $3.16 \mathrm{E}+02$ | $3.15 \mathrm{E}+02$ | $3.20 \mathrm{E}+02$ |
| $f_{24}$ | DBBO | $3.14 \mathrm{E}+02$ | $3.20 \mathrm{E}-01$ | $3.14 \mathrm{E}+02$ | $3.14 \mathrm{E}+02$ | $3.16 \mathrm{E}+02$ |
|  | LBBO | $3.15 \mathrm{E}+02$ | $5.25 \mathrm{E}-02$ | $3.15 \mathrm{E}+02$ | $3.15 \mathrm{E}+02$ | $3.15 \mathrm{E}+02$ |
|  | BBBO | $3.16 \mathrm{E}+02$ | $3.29 \mathrm{E}-01$ | $3.16 \mathrm{E}+02$ | $3.15 \mathrm{E}+02$ | $3.17 \mathrm{E}+02$ |
|  | FBBO | $2.00 \mathrm{E}+02$ | $0.00 \mathrm{E}+00$ | $2.00 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ |
|  | GSA | $2.00 \mathrm{E}+02$ | $1.72 \mathrm{E}-02$ | $2.00 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ |
|  | BBO | $2.29 \mathrm{E}+02$ | $4.59 \mathrm{E}+00$ | $2.28 \mathrm{E}+02$ | $2.24 \mathrm{E}+02$ | $2.44 \mathrm{E}+02$ |
|  | M1BBO | $2.28 \mathrm{E}+02$ | $4.34 \mathrm{E}+00$ | $2.27 \mathrm{E}+02$ | $2.24 \mathrm{E}+02$ | $2.43 \mathrm{E}+02$ |
|  | M2BBO | $2.29 \mathrm{E}+02$ | $5.30 \mathrm{E}+00$ | $2.28 \mathrm{E}+02$ | $2.24 \mathrm{E}+02$ | $2.49 \mathrm{E}+02$ |
| $f_{25}$ | DBBO | $2.33 \mathrm{E}+02$ | $7.48 \mathrm{E}+00$ | $2.30 \mathrm{E}+02$ | $2.23 \mathrm{E}+02$ | $2.46 \mathrm{E}+02$ |
|  | LBBO | $2.25 \mathrm{E}+02$ | $1.41 \mathrm{E}+00$ | $2.25 \mathrm{E}+02$ | $2.22 \mathrm{E}+02$ | $2.29 \mathrm{E}+02$ |
|  | BBBO | $2.19 \mathrm{E}+02$ | $5.77 \mathrm{E}+00$ | $2.21 \mathrm{E}+02$ | $2.07 \mathrm{E}+02$ | $2.26 \mathrm{E}+02$ |
|  | FBBO | $2.00 \mathrm{E}+02$ | $9.99 \mathrm{E}-10$ | $2.00 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ |
|  | GSA | $2.00 \mathrm{E}+02$ | $2.40 \mathrm{E}-10$ | $2.00 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ |
|  | BBO | $2.09 \mathrm{E}+02$ | $3.18 \mathrm{E}+00$ | $2.08 \mathrm{E}+02$ | $2.06 \mathrm{E}+02$ | $2.17 \mathrm{E}+02$ |
|  | M1BBO | $2.02 \mathrm{E}+02$ | $2.71 \mathrm{E}+00$ | $2.01 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ | $2.16 \mathrm{E}+02$ |
|  | M2BBO | $2.10 \mathrm{E}+02$ | $3.63 \mathrm{E}+00$ | $2.09 \mathrm{E}+02$ | $2.05 \mathrm{E}+02$ | $2.22 \mathrm{E}+02$ |
|  | DBBO | $2.02 \mathrm{E}+02$ | $6.40 \mathrm{E}-01$ | $2.02 \mathrm{E}+02$ | $2.01 \mathrm{E}+02$ | $2.04 \mathrm{E}+02$ |
|  | LBBO | $2.14 \mathrm{E}+02$ | $2.33 \mathrm{E}+00$ | $2.14 \mathrm{E}+02$ | $2.08 \mathrm{E}+02$ | $2.19 \mathrm{E}+02$ |
|  | BBBO | $2.06 \mathrm{E}+02$ | $3.49 \mathrm{E}+00$ | $2.07 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ | $2.12 \mathrm{E}+02$ |

Table 3 continued


The better results are highlighted by bold


Fig. 4 10-Dimensional Boxplots; $\mathbf{a}$ for mean error, $\mathbf{b}$ for standard deviation, $\mathbf{c}$ for median, $\mathbf{d}$ for best and $\mathbf{e}$ for worst
algorithm and ' $=$ ' sign appears if FBBO is not significantly different than compared algorithms. Out of 210 comparisons, there are 152 and $154^{\prime}+$ ' signs for 10 - and 30 -dimensional problems, respectively. Therefore, the conclusion from all analyses is that FBBO is significantly a better optimizer than other considered algorithms. The so-obtained FBBO is better in terms of accuracy which is the key improvement of the proposed algorithm.

### 5.4 Algorithm complexity

As per the suggestion of IEEE CEC 2014, the complexity of an algorithm for 10 and 30 dimension is calculated. The complexity is determined in terms of $\hat{T 2}$ and $(\hat{T} 2-T 1) / T 0$, where $T 0, T 1$ and $\hat{T 2}$ are given below:
(a) $T 0$ is the computing time for the test program given as follows:


Fig. 5 30-Dimensional Boxplots; $\mathbf{a}$ for mean error, $\mathbf{b}$ for standard deviation, $\mathbf{c}$ for median, $\mathbf{d}$ for best and $\mathbf{e}$ for worst

```
for i=1:10000 do
    x=0.55 + (double) i;
    x=x + x; x=x/2; x=x*x; x=sqrt(x); x=log(x); x=exp(x);
    x=x/(x+2);
end for
```

(b) $T 1$ is the computing time for $2 \times 10^{5}$ evaluation of $f_{18}$ of a given dimension $D$.
(c) $T 2$ is the computing time for the algorithm with $2 \times 10^{5}$ evaluations of $f_{18}$ of a given dimension $D$.
(d) Execute five $T 2$ values in step c and evaluate $\hat{T} 2=$ Mean (five $T 2$ values)

The complexity of the algorithm is evaluated in terms of $T 0, T 1, \hat{T} 2$ and $(\hat{T} 2-T 1) / T 0$ on 10 and 30 dimensions. The algorithm complexities are shown in Table 6. For clear understanding the time complexity is also shown in Fig. 6 for both 10 and 30 dimensions. This shows that FBBO has a slightly higher complexity as compared to other considered algorithms.

In case of accuracy, Tables 2 and 3 show that FBBO performs better on 21 and 22 functions out of 30 functions, in 10 and 30 -dimensional space, respectively. In Table 6, 6 algorithms (BBBO, M2BBO, M1BBO, LBBO, GSA and FBBO) have more complexity than BBO in 10-dimensional

Table 4 Mann-Whitney U rank-sum test at $\alpha=0.05$ level of significance with FBBO based on average error for 10-dimensional CEC 2014 benchmark set (TP: Test Problem)

| TP | GSA | BBO | M1BBO | M2BBO | DBBO | LBBO | BBBO |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{1}$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $f_{2}$ | + | + | + | + | + | + | + |
| $f_{3}$ | $+$ | + | + | + | + | + | $+$ |
| $f_{4}$ | $+$ | + | + | = | = | + | + |
| $f_{5}$ | $+$ | $=$ | + | $=$ | + | = | $=$ |
| $f_{6}$ | $+$ | $+$ | $=$ | $+$ | $=$ | $+$ | $+$ |
| $f_{7}$ | = | + | + | + | + | + | + |
| $f_{8}$ | $+$ | + | = | $+$ | + | - | $+$ |
| $f_{9}$ | $+$ | - | - | - | - | - | - |
| $f_{10}$ | + | + | $+$ | + | + | $=$ | + |
| $f_{11}$ | $+$ | + | + | $+$ | + | + | + |
| $f_{12}$ | - | - | $+$ | - | - | + | + |
| $f_{13}$ | $=$ | + | $+$ | $+$ | + | $+$ | + |
| $f_{14}$ | $+$ | $=$ | $+$ | $+$ | = | $+$ | $=$ |
| $f_{15}$ | $+$ | + | $+$ | + | + | + | + |
| $f_{16}$ | $+$ | = | = | $=$ | = | $=$ | + |
| $f_{17}$ | $+$ | $+$ | $+$ | $+$ | - | + | + |
| $f_{18}$ | $+$ | $+$ | $+$ | + | + | + | + |
| $f_{19}$ | $+$ | $+$ | $+$ | $+$ | + | + | $=$ |
| $f_{20}$ | $+$ | $+$ | $=$ | + | = | $=$ | $=$ |
| $f_{21}$ | $+$ | + | + | + | = | + | $+$ |
| $f_{22}$ | $+$ | $=$ | $=$ | $+$ | + | + | + |
| $f_{23}$ | $+$ | $+$ | + | $+$ | + | + | $+$ |
| $f_{24}$ | $+$ | = | $=$ | $=$ | $=$ | = | $=$ |
| $f_{25}$ | $+$ | + | + | + | + | + | + |
| $f_{26}$ | $+$ | $=$ | $=$ | $=$ | $=$ | $=$ | $=$ |
| $f_{27}$ | $+$ | + | + | + | + | + | + |
| $f_{28}$ | $+$ | $+$ | $+$ | + | + | + | $+$ |
| $f_{29}$ | $+$ | $=$ | - | $=$ | - | - | $+$ |
| $f_{30}$ | $+$ | $+$ | + | + | - | + | + |
| Total number of ' + ' signs | 27 | 21 | 21 | 22 | 17 | 21 | 23 |

of '+' signs

Table 5 Mann-Whitney U rank-sum test at $\alpha=0.05$ level of significance with FBBO based on average error for 30-dimensional CEC 2014 benchmark set (TP: Test Problem)

| TP | GSA | BBO | M1BBO | M2BBO | DBBO | LBBO | BBBO |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{1}$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $f_{2}$ | + | + | + | + | + | + | + |
| $f_{3}$ | $+$ | $+$ | + | + | $=$ | $+$ | $=$ |
| $f_{4}$ | + | $+$ | $=$ | $+$ | $+$ | $+$ | $+$ |
| $f_{5}$ | $=$ | $=$ | $=$ | $=$ | $=$ | $=$ | $=$ |
| $f_{6}$ | + | $=$ | $=$ | $=$ | $=$ | $+$ | $+$ |
| $f_{7}$ | - | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $f_{8}$ | + | $+$ | $=$ | $+$ | $+$ | - | $+$ |
| $f_{9}$ | $+$ | $=$ | + | $=$ | $=$ | $+$ | $+$ |
| $f_{10}$ | $+$ | $=$ | + | $=$ | + | $+$ | $+$ |
| $f_{11}$ | + | $+$ | $=$ | + | $=$ | $+$ | + |
| $f_{12}$ | - | $=$ | $=$ | = | $=$ | $=$ | = |
| $f_{13}$ | $=$ | + | $=$ | + | $=$ | $+$ | + |
| $f_{14}$ | = | = | = | = | $+$ | - | - |
| $f_{15}$ | $=$ | + | + | $+$ | + | $+$ | $+$ |
| $f_{16}$ | $+$ | $=$ | $=$ | = | = | $=$ | $=$ |
| $f_{17}$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $f_{18}$ | $+$ | $+$ | + | $+$ | + | $+$ | $=$ |
| $f_{19}$ | + | $+$ | $=$ | $+$ | $=$ | $=$ | $+$ |
| $f_{20}$ | + | $+$ | $+$ | + | $+$ | + | + |
| $f_{21}$ | $=$ | $+$ | $+$ | $+$ | $+$ | $+$ | + |
| $f_{22}$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $f_{23}$ | $+$ | $+$ | $+$ | + | + | $+$ | + |
| $f_{24}$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $f_{25}$ | $+$ | $+$ | + | + | + | $+$ | $+$ |
| $f_{26}$ | $+$ | $+$ | $=$ | $=$ | $=$ | $+$ | $+$ |
| $f_{27}$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $f_{28}$ | + | $+$ | $+$ | $+$ | $+$ | $+$ | $+$ |
| $f_{29}$ | $+$ | $+$ | + | $+$ | $+$ | $+$ | $+$ |
| $f_{30}$ | + | $+$ | = | $+$ | $+$ | $+$ | $+$ |
| Total number | 23 | 23 | 18 | 22 | 20 | 24 | 24 |

of ' + ' signs

## 6 Conclusion

This paper presents fireworks-inspired biogeography-based optimization (FBBO) to improve the solution diversity. FBBO uses migration and mutation operator of BBO algorithm and explosion operator of fireworks algorithms (FWA). A promising search strategy has been developed without affecting the algorithms' original efficiencies. The numerical results show that there is a scope of research in hybridizing meta-heuristics to solve complex continuous optimization problems. The proposed FBBO is a better tool to solve unconstrained nonlinear optimization problems. However, as

Table 6 Algorithm complexity (in s)

|  | $D=10$ | $D=30$ |
| :--- | :--- | :--- |
| $T 0$ | 0.9302 | 0.9302 |
| $T 1$ | 3.6979 | 4.6848 |
| $\hat{T 2}$ |  |  |
| GSA | 15.3796 | 104.3042 |
| BBO | 13.5666 | 27.2668 |
| M1BBO | 15.7457 | 133.5399 |
| M2BBO | 15.1084 | 140.9946 |
| DBBO | 11.6635 | 86.8240 |
| LBBO | 17.0572 | 132.3959 |
| BBBO | 16.0102 | 117.8927 |
| FBBO | 67.7974 | 187.8814 |
| $(\hat{T 2}-T 1) / T 0$ |  |  |
| GSA | 12.5588 | 107.0989 |
| BBO | 10.6096 | 24.2774 |
| M1BBO | 12.9523 | 138.5297 |
| M2BBO | 12.2672 | 146.5441 |
| DBBO | 8.5636 | 88.3063 |
| LBBO | 14.3623 | 137.2998 |
| BBBO | 13.2367 | 121.7077 |
| FBBO | 68.9122 | 196.9512 |



Fig. 6 Algorithm complexity (in s)
indicated by the numerical experiments, the parameter value needs to be fine tuned to obtain the best results on different problems. This hybrid approach can be applied to solve realworld optimization problems. FBBO can also be customized for constrained and multiobjective optimization problems.

Acknowledgements Author Pushpa Farswan has received research Grants from South Asian University, India.

## Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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